LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2023**

UMT 6501 – COMPLEX ANALYSIS

Date: 29-04-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART – A

Q. No **Answer ALL questions** **(10 x 2 = 20 Marks)**

1 Evaluate: $\lim_{z\to 3} \frac{z^2-9}{z-3}$.

2 Show that the function $f(z) = e^x(\cos y - i \sin y)$ is nowhere differentiable.

3 State Cauchy-Goursat theorem.

4 Find a so that $u(x, y) = ax^2 - y^2 + xy$ is harmonic.

5 Expand $f(z) = \frac{1}{1-z}$ as a Taylor's series about the point $z = 0$.

6 Evaluate $\int_C \frac{dz}{z-3}$ where C is the circle$|z - 2| = 5$.

7 What is the nature of the singularity $z = 0$ of the function$f(z) = \frac{\sin z - z}{z^3}$.

8 Calculate the residue of $f(z) = \frac{e^{2z}}{(z+1)^2}$ at its pole.

9 Define Bilinear transformation.

10 Define conformal mapping.

PART – B

Answer any FIVE questions **(5 x 8 = 40 Marks)**

11 Verify C-R equations for $f(z) = \begin{cases} \frac{xy}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$. Is it differentiable at the origin? (8)

12 Show that an analytic function in a region with constant modulus is constant. (8)

13 Evaluate $\int_C \frac{z\,dz}{z^2-1}$ where C is $|z| = 2$, using Cauchy's integral formula. (8)

14 State Liouville's theorem and deduce the fundamental theorem of algebra. (8)

15 State and prove Cauchy residue theorem. (8)

16 Determine the bilinear transformation that maps the points $-1,0,1$ in the $z-$plane onto the points $0, i, 3i$ in the $w-$plane. (8)

17 Find the residue of $f(z) = \frac{\sin z}{z \cos z}$ at each of its poles inside the circle$|z| = 2$. (8)

18 Expand $f(z) = \frac{-1}{(z-1)(z-2)}$ as a Laurent series in$1 < |z| < 2$. (8)

PART – C

Answer any TWO questions **(2 x 20 = 40 Marks)**

19 State and prove necessary and sufficient condition for $f(z)$ to be differentiable at a point. (20)

20 (a) If $f(z) = u + iv$ is an analytic function and $u(x, y) = \frac{sin2x}{cosh2y+cos2x}$,find $f(z)$ (8)

(b) State and Prove Cauchy's integral formula (12)

21 (a) State and prove Taylor's theorem. (12)

(b) Using Contour integration, evaluate $\int_0^{2\pi} \frac{d\theta}{5+4\sin\theta}$. (8)

22 (a) Discuss the transformation $\omega = \frac{1}{z}$ (10)

(b) State and prove Rouche's theorem. (10)

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