LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

THIRD SEMESTER – NOVEMBER 2007

MT 3500 - ALGEBRA, CALCULUS & VECTOR ANALYSIS

Date : 27/10/2007 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

Answer all the questions: **10 x 2 = 10**

1.Form a partial differential equation by eliminating the arbitrary constants from the equation

$z = (x^2 + a)(y^2 + b).$

2. Show that $\nabla . \vec{r} = 3$.

3. Evaluate $\int_0^1\int_0^2 (x^2 + y^2)\, dy\, dx.$

4. Find the value of a if $(axy - z)\vec{i} + (a - 2)x^2\vec{j} + (1 - a)xz^2\vec{k}$ is irrotational .

5. If $\vec{f} = x^2\vec{i} + xy\vec{j}$ find $\int_C \vec{f}.d\vec{r}$ from (0,0) to (1, 1) along $y = x$.

6. Show that $\partial(u,v) / \partial(u,v) = 1$.

7. Show that $\beta(m, n) = \beta(n, m)$.

8. State Wilson's theorem for real numbers.

9. Among gradφ, divφ and curlφ which is a scalar function ?

10 Find the value of L [e^{-at}].

Answer any five questions: **5 x 8 = 40**

11. Find the directional derivative of the function $Q = xy + yz + zx$ at the point (1, 2, 0) in the

direction of $\vec{i} + 2\vec{j} + 2k$.

12 .State and prove and Cauchy's inequality for R^n.

13. Derive the relation between Beta and Gamma functions.

14. Solve using Charpit's method $z^2(p^2 z^2 + q^2) = 1$, where $p = \partial z / \partial x$ and $q = \partial z / \partial y$.

15. Verify Stoke's theorem for the function $\vec{F} = x^2\vec{i} + xy\vec{j}$, integrated along the region $z = 0$,

$x = 0$ to a and $y = 0$ to a.

16. Using Lagrange's method, solve $x(y-z)p + y(z-x)q = z(x-y)$,

where $p = \partial z/\partial x$ and $q = \partial z/\partial y$.

17. Show that (i) $\vec{\nabla} \times \nabla \varphi = 0$

(ii) $\nabla \times (\vec{A} \times \vec{B}) = \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A}) + (\vec{B} \cdot \nabla)\vec{A} - (\vec{A} \cdot \nabla)\vec{B}$

18. State and prove Fermat's theorem.

Answer any two questions: **2 x 20 =40**

19. Find the value of (i) L [sinhat]

(ii) $L[\cos^2 6t]$

(iii) $L^{-1}[s/(s^2+a^2)^2]$

(iv) $L^{-1}[(s+2)/(s^2+4s+5)^2]$.

20.(i)Evaluate $\int\int_R (x-y)^4 e^{x+y} dx\, dy$ where R is the square with vertices(1,0),(2 ,1),(1, 2) , and (0,1),where $x + y = 1$, $x + y = 3$, $x - y = 1$ and $x - y = -1$, by changing the variables .

(ii) Evaluate $\int\int_R x\, y\, (1-x-y)^{1/2} dx\, dy$ where R is the triangle with sides $x + y = 1$, $x = 0$, $y = 0$, if $x + y = u$, $y = uv$.

21. Solve the partial differential equations

(i) $p^2 + q^2 = 1$

(ii) $z = px + qy + p^2q^2$

(iii) $p - x^2 = q + y^2$

22. (i)Verify Green's theorem in the xy plane for $\int_C (3x^2 - 8y^2)\, dx + (4y - 6xy)\, dy$, where C is a curve given by $x = 0$, $y = 0$ and $x + y = 1$.

(ii) Find the highest power of 5 in 1800**!**

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