LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **MATHEMATICS**

FIRST SEMESTER – NOVEMBER 2011

MT 1811 - REAL ANALYSIS

Date : 03-11-2011 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

Answer all the questions.

I. a)(1) Prove that the existence of Riemann-Steiltjes integral may be knocked down even if the integrand is altered at just one point.

OR

a)(2) Prove that neither the existence nor the value of a Riemann integral is affected if the integrand is altered at finite number of points.

(5)

b) Define Step function and proving the necessary results prove that every finite sum can be written as a Riemann – Stieltjes integral.
(15)

OR

c) Let α be an increasing function on [a,b]. Prove that the following statements are equivalent.

(i) f is Riemann -Stieltjes integrable over [a,b]

(ii) f satisfies the Riemann's condition

(iii) The upper integral value of f is equal to the lower integral value over [a,b].
(15)

II. a)(1) Define $f_n(x) = n^2 x e^{-nx^2}$ for all x∈ [0,1]. Verify whether the limit operation and integral operation could be interchanged.

OR

a)(2) Prove that in the case of uniform convergence of a finite sequence of continuous functions to a limit function, the limit function will also be continuous.

(5)

b)(1) State the Cauchy condition for uniform convergence of series and also the Weierstrass M test.

b)(2) Let α be of bounded variation on [a,b]. Assume that each term of the sequence $\{f_n\}$ is a real-valued function such that $f_n \in R(\alpha)$ on [a,b] for each n =1,2,3,… Assume that $f_n \to f$ uniformly on [a,b] and define $g_n(x) = \int_a^x f_n(t)\,d\alpha(t)$ if x ∈ [a,b] , n =1,2,3,… Then prove that

(i) f ∈ R(α) on [a,b].

(ii) $g_n \to g$ uniformly on [a,b] where $g(x) = \int_a^x f(t)\,d\alpha(t)$

(5+10)

OR

c) Assume that each term of $\{f_n\}$ is a real-valued function having a finite derivative at each point of an open interval (a,b). Assume that for at least one point x_o in (a,b) the sequence $\{f_n(x_o)\}$ converges. Assume further that there exists a function g such that $f_n' \rightarrow g$ uniformly on (a,b). Then prove that (i) there exists a function f such that $f_n \rightarrow f$ uniformly on (a,b)

(ii) For each x in (a,b) the derivative f '(x) exists and equals g(x).

(15)

III. **a)(1)** State and prove Parseval's formula.

OR

a)(2) State the two major problems related to the convergence and representation of Trignometric series and mention a few famous mathematicians who have analyzed techniques to resolve these issues.

(5)

b)(1) State and prove Riesz- Fischer's theorem.

b)(2) State and prove Riemann- Lebesgue Lemma

(8+7)

OR

c)(1) State Jordan's and Dini's theorems.

c)(2) State and prove Riemann Localization theorem.

(4+11)

IV. **a)(1)** State the sufficient condition for differentiability of a multivariate function and the statement for its equality of mixed partial derivatives.

OR

a)(2) Give an example a function which has finite directional derivative **f' (c;u)** for every **u** but may fail to be continuous at **c**.

(5)

b)(1) Let S be an open connected subset of R^n and let **f:** S→$\mathbf{R^m}$ be differentiable at each point of S. If **f '(c) = 0** for each **c** in S, then prove that **f** is constant on S.

b)(2) Assume that one of the partial derivatives D_1**f**, D_2**f**,…, D_n**f** exists at **c** and that the remaining (n-1) partial derivatives exists in some n-ball **B(c)** and are continuous at **c**. Then prove that **f** is differentiable at **c**.

(5+10)

OR

c) Proving all the necessary results, prove that if both the partial derivatives D_r**f** and D_k**f** exist in a n-ball **B(c)** and if both $D_{r,k}$ **f** and $D_{k,r}$ **f** are continuous at **c** then $D_{r,k}$ **f(c) =** $D_{k,r}$ **f(c).**

(15)

V. **a)(1)** Let A be an open subset of $\mathbf{R^n}$ and assume **f**:A →$\mathbf{R^n}$ is continuous and has finite partial derivatives $D_j f_i$ on A. If $\mathbf{J_f}$ **(x)** ≠ 0 for all **x** in A, then (stating the necessary theorems) prove that **f** is an open mapping.

OR

a)(2) **L**et A be an open subset of $\mathbf{R^n}$ and assume **f**:A →$\mathbf{R^n}$ is continuous and has finite partial derivatives

$D_j f_i$ on A. If **f** is one to one on A and if $\mathbf{J_f(x)} \neq 0$ for each **x** in A, then prove that **f(A)** is open.

(5)

b) State and prove the Inverse Function theorem.

OR

c) (1) State Implicit Function theorem.

c)(2) Assume that **f** = $(f_1, f_2, \ldots f_n)$ has continuous partial derivatives $D_j f_i$ on an open set S in $\mathbf{R}^n$ and that the Jacobian determinant $\mathbf{J_f(a)} \neq 0$ for some point **a** in S. Then prove that there is an n- ball B(**a**) on which **f** is one – to - one.

(5+ 10)

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