LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – NOVEMBER 2011

MT 5505/MT 5501 - REAL ANALYSIS

Date : 31-10-2011 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

PART-A

ANSWER ALL QUESTIONS **(10×2=20)**

1. Give an example of a subset of real numbers which is not order complete.
2. Write the triangular inequality.
3. Define interior of a subset of metric spaces.
4. Give an example of a compact subset of set of real numbers.
5. Show that every convergent sequence is a Cauchy sequence.
6. Define complete metric space and give an example of a space which is not complete.
7. Show that a function which is differential at c is also continuous at c.
8. Define local minimum and local maximum of a function at a point.
9. Define monotonic sequence of real numbers and give an example of it.
10. Give an example of a function which is not Riemann – Stieltjes integrable.

PART –B

ANSWER ANY FIVE QUESTIONS **(5×8= 40)**

11. State and prove Cauchy Schwarz inequality.
12. Show that the set of all sequences whose terms are 0 and 1 is uncountable.
13. Show that every nonempty open subset S of R^1 is the union of a countable collection of pair wise disjoint open intervals whose end points do not belong to S.
14. Show that every compact subset of a metric space is complete.
15. If f is an increasing function on [a,b] and c ∈ (a, b), show that f(c+) and f (c−) exist and f(c−)≤ f(c) ≤ f(c+).
16. State and prove chain rule for differentiation.
17. If f, g ∈R(α) on [a,b] show that for constants a and b, af+bg∈ R(α) and $\int_a^b (af+bg)\,d\alpha = a\int_a^b f\,d\alpha + b\int_a^b g\,d\alpha$.
18. If f∈R (α) on[a,b] show that α ∈ R(f) on [a,b] and $\int_a^b f\,d\alpha + \int_a^b \alpha\,df = f(b)\alpha(b) - f(a)\alpha(a)$.

PART – C

Answer any TWO questions **(2×20=40)**

19. (a) If Γ is a countable collection of pair wise disjoint countable sets, show that $\bigcup_{F\in r} F$ is also countable.
    (b) If A is a countable set and B is uncountable show that B – A is similar to B.
    (c) State and prove Minkowski's inequality. (6+6+8)

20. (a) Show that a subset E of a metric space (M, d) is closed if and only if it contains all its adherent points.
    (b) State and Prove Bolzano Weirstrass theorem. (10 +10)

21. (a) Show that Euclidean space R^n is complete.
    (b) Show that continuous function defined on compact space is uniformly continuous. (10 + 10)

22. (a) State and prove Taylors theorem.
    (b) Let f∈R(α)on [a,b], α is differentiable on [a,b] and α' is continuous on [a,b]. Show that Riemann integral $\int_a^b f\alpha' dx$ exists and $\int_a^b f\, da = \int_a^b f\alpha' dx$. (10+10)

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