LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2013**

MT 3810 - TOPOLOGY

Date : 05/11/2013 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

Answer ALL the questions. Each question carries 20 marks.

I. a)1) Let X be a metric space with metric d. Show that d_1 defined by $d_1(x,y) = \frac{d(x,y)}{1+d(x,y)}$ is also a metric

on X.

OR

a)2) Let X be a metric space. Prove that any union of open sets in X is open and any finite intersection of open sets in X is open.

(5)

b)1) If a convergent sequence in a metric space has infinitely many distinct points, then prove that its limit is a limit point of the set of points of the sequence.

b)2) Let X be a complete metric space and Y be a subspace of X. Prove that Y is complete if and only if it is closed.

b)3) State and prove Cantor's intersection theorem.

(4+6+5)

OR

c)1) Let X and Y be metric spaces and f, a mapping of X into Y. Then prove that f is continuous if and only if $f^{-1}(G)$ is open in X, whenever G is open in Y.

c)2) Let f: X→ Y be a mapping of one topological space into another.

Show that f is continuous ⇔ $f^{-1}(F)$ is closed in X whenever F is closed in Y.

⇔ $f(\overline{A}) \subseteq \overline{f(A)}$ for every subset A of X.

(5+10)

II. a)1) State Kuratowski's closure axioms.

OR

a)2) Prove that a topological space is compact if every basic open cover has a finite subcover.

(5)

b)1) State and prove Lindelof's theorem.

b)2) State and prove Tychonoff's theorem.

(7+8)

OR

c) State and prove Ascoli's theorem.

(15)

III. a)1) Prove that every subspace of a Hausdorff space is a Hausdorff space.

OR

a)2) Show that every compact Hausdorff space is normal.

(5)

b)1) In a Hausdorff space, show that any point and a disjoint compact space can be separated by open sets.

b)2) Prove that every compact subspace of a Hausdorff space is closed.

b)3) Show that a one-one continuous mapping of a compact space onto a Hausdorff space is a homeomorphism. (7+ 4 + 4)

OR

c) State and prove Urysohn imbedding theorem. (15)

IV. a)1) Prove that any continuous image of a connected space is connected.

OR

a)2) Show that the range of a continuous real function defined on a connected space is an interval. (5)

b)1) Prove that a subspace of the real line R is connected ⇔ it is an interval. In particular, show that R is connected.

b)2) Prove that the spaces ⊔ ^{n} and ⊔ ^{n} are connected. (8+7)

OR

c)1) Define totally disconnected space. Let X be a Housdorff space. If X has an open base whose sets are also connected, then prove that X is totally disconnected.

c)2) Let X be a compact Housdorff space. Then prove that X is totally disconnected ⇔ it has an open base whose sets are also closed. (7+8)

V. a)1) State the two equivalent forms of Weierstrass theorem.

OR

a)2) Prove that X_∞ is Housdorff. (5)

b)1) Prove the following lemma: Let X be a compact Housdorff space with more than one point and let L be a closed sublattice of ⊔ $(X, \ )$ with the following property: if x and y are distinct points of X and a and b are any two real numbers, then there exists a function f in L such that f(x) =a and f(y)= b. Then L equals ⊔ $(X, \ )$.

b)2) Prove the following lemma: Let X be an arbitrary topological space. Then every closed subalegbra of ⊔ $(X, \ )$ is also a closed sublattice of ⊔ $(X, \ )$. (7+8)

OR

c) State and prove Real Stone Weierstrass theorem. (15)

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