LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2013**

MT 3875 - MATHEMATICAL FINANCE MODELS

Date : 15/11/2013 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

Answer **ALL** Questions. All questions carry equal marks.

1. (a) (i) Define i) Geometric Brownian motion process. ii) Brownian motion process. (5)

OR

(ii) If an amount is deposited at some interest rate *r* compounded annually, in how many years the amount will be doubled. (5)

(b) (i) Obtain Geometric Brownian motion process as a limit of simple models. (15)

OR

(ii) If Adam borrow Rs.1000 for one year at an interest rate of 8% per year, Calculate the interest paid by him i) annually ii) half yearly iii) Quarterly iv) monthly, Which of the term period is more profitable for him.

(iii) Find the rate of return of a two year investment that for an initial payment of 1000, gives a return at the end of the first year of 500 and a return at the end of the second year of: a) 300, b) 500, c) 700. (8+7)

2. (a)(i) Let the initial price of the stock be 100. Price after one period is assumed to be either 200 or 50. Find the probability p that results in a 0 expected return for purchasing the stock. (5)

OR

(ii) Prove that one should never exercise an American style call option before its expiration time t. (5)

(b) (i) State and prove Arbitrage theorem. (15)

OR

(ii) For two investments, first of which costs the fixed amount C_1 and the second fixed price amount C_2. If the present value from the first investment is always identical to that of the second investment, then show that either $C_1 = C_2$ or there is an arbitrage. (15)

3. (a) (i) Obtain E[IS(t)] used in the Black- Scholes Formula.

OR

(ii) Prove that the dividend for each share of the security is paid continuously in time at a rate equal to a fixed fraction f of the price of the security. (5)

(b) (i) The price of a certain security follows a geometric Brownian motion with $\mu = .05$ and $\sigma = 0.3$. The present value of the security is 95. If the interest rate is 4%, find the no arbitrage cost of a call option that expires in 3 months and has exercise price 100.

(ii) Show that $C(s,t,k,\sigma,r)$ is decreasing in *k*. (10+5)

OR

(iii) Briefly explain the Valuing investments by expected utility. (15)

4. (a) (i) Derive the mean and variance of lognormal distribution. (5)

OR

(ii) Derive the value of β_i in capital assets pricing model. (5)

(b) (i) Explain the procedure of estimating σ when i) Closing prices are given. ii) Closing and opening prices are given. iii) Closing, opening, high, low data values are given. (15)

OR

(ii) Given three investment projects with the following return functions i) $f_1(x)=\frac{10x}{1+x}, x=0,1,...$ ii) $f_2(x)=\sqrt{x}, x=0,1,...$ iii) $f_3(x)=10(1-e^{-x}), x=0,1,...$ what will be the maximum return for 5 investments? (15)

5. (a) (i) Explain value at risk (VAR). (5)

OR

(ii) Explain barrier call option with a specified strike price. (5)

(b) (i) If one has to invest 100 in 2 securities whose rates of return have the following expected values and standard deviations $r_1=0.15, v_1=0.2, r_2=0.18, v_2=0.25$ and correlation co efficient between returns is $\rho=-0.4$. Find the optimal portfolio when using $v(x)=1-e^{-0.005x}$. Obtain the expected optimal utility. (15)

OR

(ii) Derive the pricing Exotic options by simulation. (15)

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