

LOYOLA COLLEGE (AUTONOMOUS) CHENNAI 600 034
B. Sc DEGREE EXAMINATION-Physics
First Semester-November 2014
MT 1100- Mathematics for Physics

Time: Forenoon/Afternoon

Max: 100 Marks

Date: / /2014

PART A

Answer ALL the questions

(10 x 2 =20)

1. If $y = \cos(ax + b)$, find y_n .
2. Find the polar subtangent and subnormal of the curve $r = e^{\theta \cot \alpha}$.
3. Prove that $\frac{e^2 - 1}{e^2 + 1} = \frac{\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \dots}{1 + \frac{1}{2!} + \frac{1}{4!} + \dots}$.
4. Show that $\log \frac{a+x}{a-x} = \frac{2ax}{a^2+x^2} + \frac{1}{3} \left(\frac{2ax}{a^2+x^2} \right)^3 + \frac{1}{5} \left(\frac{2ax}{a^2+x^2} \right)^5 + \dots$.
5. Find the Laplace transform of $t^3 + t + 2$.
6. Find $L^{-1} \left(\frac{1}{(s-3)^5} \right)$.
7. Write the expansion of $\tan \theta$.
8. Prove that $\cosh^2 x - \sinh^2 x = 1$.
9. Two unbiased dice are thrown. Find the probability that the total of the numbers on the dice is greater than 8.
10. What is the chance that a leap year selected at random will contain 53 Sundays?

PART B

Answer any FIVE questions

(5 x 8 =40)

11. Find the n^{th} derivative of (i) $\frac{7x-1}{(3x-1)(2x-1)}$ and (ii) $\log(a^2x^2 - b^2)$.
12. Find the angle of intersection of the cardioids $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$.
13. Find the sum to infinity of the series $\frac{3}{4} + \frac{3.5}{4.8} + \frac{3.5.7}{4.8.12} + \dots$.
14. Verify Cayley- Hamilton theorem for the matrix $\begin{pmatrix} 1 & 3 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix}$.
15. Find the transform of the rectangular wave given by the function $f(t) = \begin{cases} 1, & 0 < t < b \\ -1, & b < t < 2b \end{cases}$ and $f(t + 2b) = f(t)$.
16. Write down the expansion of $\cos 6\theta$ in terms of $\cos \theta$.

17. If $\sin(A + iB) = x + iy$, prove that $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$ and $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$.

18. A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which (i) neither car is used, and (ii) the proportion of days on which some demand is refused.

PART C

Answer any TWO questions

(2 x 20 = 40)

19. (a) If $y = (x + \sqrt{1 + x^2})^m$, prove that $(1 + x^2)y_2 + xy_1 = m^2 y$ and $(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

(b) Find the maximum and minimum values of the function $f(x) = x^3 - 18x^2 + 96x + 4$.

(10+10)

20. (a) Find the sum to infinity of the series $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \frac{1+3+3^2+3^3}{4!} + \dots$.

(b) If a, b, c denote three consecutive integers, show that $\log_e b = \frac{1}{2} \log_e a + \frac{1}{2} \log_e c + \frac{1}{2ac+1} + \frac{1}{3} \left(\frac{1}{2ac+1}\right)^3 + \dots$.

(10+10)

21. (a) Find the inverse Laplace transform of $\frac{5s+3}{(s+1)(s^2+2s+2)}$.

(b) If $\tan(x + iy) = u + iv$, prove that $\frac{u}{v} = \frac{\sin 2x}{\sinh 2y}$.

(10+10)

22. (a) Expand $\sin^3 \theta \cos^5 \theta$ in terms of sines of multiples of θ .

(b) Calculate the mean and standard deviation for the following table giving the age distribution of 542 members.

Age in years	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of Members	3	61	132	153	140	51	2

(10+10)
