LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2014**

MT 1819 - PROBABILITY THEORY & STOCHASTIC PROCESSES

Date : 10/11/2014 Dept. No. Max. : 100 Marks

Time : 01:00-04:00

Section – A

Answer all the questions 10 x 2 = 20 marks

1. Write the sample space for tossing three fair coins simultaneously.
2. Define distribution function of a random variable.
3. If $f(x) = e^{-x}$, $0<x<\infty$, zero elsewhere , find E(X).
4. If P(A) = ½ , P(B) = ¼ and $P(A\cap B) = 1/6$ find (i) P(A|B) and (ii) $P(A|B^C)$.
5. Enlist any four properties of normal distribution.
6. Write the pdf and MGF of exponential distribution .
7. Define convergence in distribution.
8. Write the sufficient conditions for a consistent estimator.
9. Write a note on testing of hypothesis.
10. Define communication of states of a Markov chain.

Section – B

Answer any Five questions 5x8 = 40 marks

11. (a) State and prove Bayes' theorem.
    (b) A problem in statistics is given to three students . Their probabilities of solving it are respectively ½ , ¼ and 1/6.They try to solve the problem independently. What is the probability that the problem will be solved ? (5 +3) marks

12. If X has the pdf $f(x) = \frac{3}{4} x (2-x)$, $0\leq x \leq 2$, zero elsewhere , find β_1 and β_2.

13. Derive the Cumulant generating function of Poisson distribution and hence find mean and variance.

14. (a) Define rectangular distribution.

    (b) If X has a uniform distribution in [0,1], find the pdfof -2 logX. Identify the distribution also.

    (2+6) marks

15. State and prove Rao-Blackwell theorem.

16. In random sampling from normal population N(μ , σ^2) , find the maximum likelihood estimators for

(i) μ when σ^2 is known (ii) σ^2 when μ is known.

17. The heights of 10 males of a given locality are found to be 70 , 67, 62, 68 , 61 , 68 , 70 , 64 ,64 , 66 inches. Is it reasonable to believe that the average height is greater than 64 inches ? Use α =0 .05.

18. (a) Explain the transition probability matrix of a Markov chain.

(b) Show that communication is an equivalence relation. (4+4) marks

Section – C

Answer any two questions 2 x 20 = 40 marks

19. (a) State and prove Boole's inequality.

(b) If X has the pdf f(x) = (3 +2x) / 18 , $2 \leq x \leq 4$, zero elsewhere find the standard deviation and mean deviation from mean. (10 +10) marks

20. (a) Obtain the first and second central moments of Beta distribution of II kind.

(b) If X is a normal variate with mean 30 and standard deviation 5 , find

(i) P($26 \leq X \leq 40$) (ii) P($X \geq 45$) (iii) P($|X-30| > 5$) (10+10) marks

21. (a) If X_1 and X_2 has the joint pdf $f(x_1, x_2) = 2$, $0 < x_1 < x_2 < 1$, zero elsewhere , find the conditional mean and variance of X_1 given $X_2 = x_2$, $0 < x_2 < 1$.

(b) State and prove Cramer –Rao inequality. (10+10) marks

22. (a) A set of 8 symmetrical coins was tossed 256 times and the frequencies of throws observed were as follows:

Number of heads :	0	1	2	3	4	5	6	7	8
Frequency of throws:	2	6	24	63	64	50	36	10	1

Fit a binomial distribution and test the goodness of fit at α =0.01.

(b) Derive forward and backward Kolmogorov differential equations of birth and death process.

(10+10) marks

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