LOYOLA COLLEGE (AUTONOMOUS) CHENNAI 600 034

M. Sc DEGREE EXAMINATION – Mathematics

Third Semester – November 2013

MT 3875 – Mathematical Finance Model   Max: 100 Marks

Time: Forenoon/Afternoon   Date:

Answer **ALL** Questions. All question carry equal marks.

1. (a) (i) Distinguish between Brownian motion and Geometric Brownian motion.

OR

(ii) If Adam borrow Rs.1000 for one year at an interest rate of 8% per year, Calculate the interest paid by him i) annually ii) half yearly iii) Quarterly iv) monthly, Which of the term period is more profitable for him. (5)

(b) (i) State and Prove Arbitrage theorem. (15)

OR

(ii) Suppose that you are to receive payments at the end of each of the next five years. Which of the following three payment sequence is preferable?

A: 12, 14, 16, 18, 20

B: 16, 16, 15, 15, 15

C: 20, 16, 14, 12, 10

(iii) Derive the yield curve equation of a continuously varying rate. (9+6)

2. (a)(i) Prove that the No arbitrage option cost C is increasing in the initial price s.

OR

(ii) Define Stocks, Shares, call option and put option with an example each. (5)

(b) (i) Explain the Properties of Black Scholes option cost Formula.

OR

(ii) For two investments, first of which costs the fixed amount C_1 and the second fixed price amount C_2. If the present value from the first investment is always identical to that of the second investment, then prove that either $C_1 = C_2$ or there is an arbitrage. (15)

3. (a) (i) Suppose that a security is presently selling for a price of 60, the nominal rate is 9% (with the unit of time being one year) and the security's volatility is 0.35. Find the no arbitrage cost of a call option that expires in three months and has a strike price 68.

OR

(ii) Suppose an investor with capital x can invest any amount between 0 & x ; if y is invested, then y is either won or lost, with respective probabilities p and 1-p. If p>1/2, how much should be invested by an investor having a log utility function? (5)

(b) (i) Assuming a Log Normal Distribution for the size of a jump, prove that ,

$$\text{No – arbitrage cost} = \sum_{n=0}^{\infty} e^{-\lambda t E[J]} \frac{(\lambda t E[J])^n}{n!} C(s,t,K,\sigma(n),r(n)) .$$

OR

(iii) Prove that in call options on dividend paying securities, for each share owned, a fixed amount D is to be paid at time t_d. (15)

4. (a) (i) Explain in detail, the Value at Risk.

OR

(ii) Derive the value of β_i in capital assets pricing model. (5)

(b) (i) Estimate the volatility parameter when the collection prices follow Geometric Brownian motion using Opening and Closing data.

OR

(ii) Given three investment projects with the following return functions

1) $f_1(x) = \frac{10x}{1+x}, x = 0,1,...$ 2) $f_2(x) = \sqrt{x}, x = 0,1,...$ 3) $f_3(x) = 10(1-e^{-x}), x = 0,1,...$

when we will get maximum returns, for we have 5 to invest. (15)

5. (a) (i) Explain the Gambling model with Unknown Win Probabilities.

OR

(ii) Explain barrier call option with a specified strike price. (5)

(b) (i) Derive the pricing Exotic options by simulation.

OR

(ii) Derive the Expectation and Variance of Present value gain by using Mean Variance analysis of Risk Neutral Priced call option. (15)

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(b) State and Prove Arbitrage theorem. (15)

2) (a) Explain the Properties of Black Scholes option cost Formula. (15)

(b) Prove that the No arbitrage option cost C is increasing in the initial price s. (5)

3) (a) Assuming a Log Normal Distribution for the size of a jump, prove that ,

$$\text{No – arbitrage cost} = \sum_{n=0}^{\infty} e^{-\lambda t E[J]} \frac{(\lambda t E[J])^n}{n!} C(s,t,K,\sigma(n),r(n))$$

(15)

(b) (b)Suppose that a security is presently selling for a price of 60, the nominal interest rate is 9%(with the unit of time being one year) and the security's volatility is .35. Find the no arbitrage cost of a call option that expires in three months and has a strike price of 68. (5)

4) (a) Estimate the volatility parameter when the collection prices follow Geometric Brownian motion using Opening and Closing data. (15)

(b) Explain in detail, the Value at Risk. (5)

5) (a) Derive the Expectation and Variance of Present value gain by using Mean Variance analysis of Risk Neutral Priced call option. (15)

(b) Explain the Gambling model with Unknown Win Probabilities. (5)