



Date: 09-11-2016
Time: 01:00-04:00

Dept. No.

Max. : 100 Marks

Part A

Answer ALL Questions:

(10 x 2 = 20)

1. Define orthogonal matrix.
2. Find the value of $\begin{vmatrix} 3 & 2 & 1 \\ 4 & 1 & -7 \\ 0 & 3 & 4 \end{vmatrix}$.
3. Express the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 4 & 3 \end{bmatrix}$ as a sum of symmetric and skew-symmetric matrix.
4. Find the rank of $A = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$.
5. Write the differential coefficient of e^x and $\sin ax$.
6. Differentiate $4x^2 - 9x - 3$ with respect to x .
7. Prove that the function $f(x) = x^3 - 3x^2 + 6$ is positive for all values of $x = 2$.
8. For what value of x is the curve $y = 3x^2 - 2x^3$ convex upwards?
9. Evaluate $\int_0^{\frac{\pi}{2}} \cos^2 \frac{x}{2} dx$.
10. Integrate: $\int \sin^4 x dx$.

Part B

Answer any FIVE Questions:

(5 x 8 = 40)

11. Show that $\begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} b & q \\ p & c \end{vmatrix} + \begin{vmatrix} p & d \\ a & q \end{vmatrix} = 0$.
12. Find the rank of the matrix $\begin{pmatrix} 1 & 1 & -3 & -1 \\ 4 & -2 & 6 & 8 \\ 15 & -3 & 9 & 21 \end{pmatrix}$.
13. (a) Find the differential coefficient of $\left(\frac{x}{x^2+1}\right)^n$.
(b) If $y = \sin x \sin 2x \sin 3x \sin 4x$, find $\frac{dy}{dx}$. (4 + 4)
14. Find the n^{th} differential coefficient of $\sin^3 \theta \cos^5 \theta$.
15. Show that for $x > 0, x - \frac{1}{2}x^2 < \log(1 + x) < x$.
16. Find the points of inflexion on the cubic $y = \frac{a^2x}{x^2+a^2}$ and show that they lie on a straight line.
17. Evaluate: $\int \frac{x^{24}}{x^{10}+1} dx$.
18. Integrate: $\int \frac{3x+1}{(x-1)^2(x+3)} dx$.

Part C

Answer any TWO Questions:

(2 x 20 = 40)

19. (a) Examine consistency and hence solve $5x - 6y + 4z = 15, 7x + 4y - 3z = 19, 2x + y + 6z = 46$.

(b) Find the Eigen values and Eigen vectors of $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$. (10 + 10)

20. (a) Verify Cayley Hamilton theorem $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.

(b) (i) Find y_n when $y = \frac{x^2}{(x-1)^2(x+2)}$.

(ii) Differentiate $e^{\sin^{-1} x}$ with respect to $\sin^{-1} x$. (10 + 7 + 3)

21. (a) If $y = \sin(m \sin^{-1} x)$, prove that $(1 - x^2)y_2 - xy_1 + m^2y = 0$ and $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

(b) Find the maximum or minimum values of $x^3y^2(6 - x - y)$. (10 + 10)

22. (a) Evaluate $I = \int_0^{\frac{\pi}{2}} \log \sin x \, dx$.

(b) Prove that $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{\frac{3}{2}}}{(\sin x)^{\frac{2}{3}} + (\cos x)^{\frac{2}{3}}} dx = \frac{\pi}{4}$. (10 + 10)
