



**LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034**

**B.Sc. DEGREE EXAMINATION – MATHEMATICS**

FIRST SEMESTER – NOVEMBER 2016

**MT 1503 - ANALYTICAL GEOMETRY OF 2D, TRIG. & MATRICES**

Date: 07-11-2016  
Time: 01:00-04:00

Dept. No.

Max. : 100 Marks

**PART – A**

**Answer all questions:**

**(10 X 2 = 20)**

1. Write the expansion of  $\tan n\theta$  in terms of  $\tan \theta$
2. If  $x = \cos \theta + i \sin \theta$  and  $\frac{1}{x} = \cos \theta - i \sin \theta$  then find  $x^n + \frac{1}{x^n}$  and  $x^n - \frac{1}{x^n}$
3. Show that  $\cosh^2 x + \sinh^2 x = \cosh 2x$
4. Prove that  $\tanh^{-1} x = \frac{1}{2} \log_e \left( \frac{1+x}{1-x} \right)$
5. Define symmetric matrix and give an example.
6. Find the eigen values of  $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$
7. Find the pole of the line  $2x + 3y + 4 = 0$  with respect to the parabola  $y^2 = 8x$
8. Define conjugate of two diameters of an ellipse.
9. Find the asymptotes of the hyperbola  $2x^2 + 2xy - 3x + y = 0$
10. Find the distance between the points  $(r_1, \theta_1)$  and  $(r_2, \theta_2)$

**PART – B**

**Answer any five questions:**

**(5 X 8 = 40)**

11. Expand  $\sin^6 x$  in a series of cosines of multiples of  $x$ .
12. Evaluate  $\lim_{x \rightarrow \pi/2} \frac{\sin x + \cos 2x}{\cos^2 x}$
13. Prove that  $\cosh 2y + \cos 2x = 2$  if  $\cos(x + iy) = \cos x + i \sin x$
14. If  $\cosh u = \sec \theta$ , prove that  $u = \log \tan \left( \frac{\pi}{4} + \frac{\theta}{2} \right)$
15. Using Cayley Hamilton theorem find the inverse of the matrix  $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$

16. If chords of a parabola are drawn through a fixed point, then show that the locus of the middle points is another parabola.
17. Find the locus of the mid-points of chords of the parabola which subtend a right angle at the vertex of the parabola.
18. If  $e, e_1$  are the eccentricities of a hyperbola and its conjugate, show that  $\frac{1}{e^2} + \frac{1}{e_1^2} = 1$

**PART – C**

**Answer any two questions:**

**(2 X 20 = 40)**

19. (i) Express  $\cos 8\theta$  in terms of  $\sin \theta$ .
- (ii) Expand  $\sin^3 \theta \cos^5 \theta$  in a series of sines of multiples of  $\theta$ . (10+10)
20. (i) Find the real part and imaginary part of  $\tan^{-1}(x + iy)$ .
- (ii) Prove that  $2e^{2A} = \cosh 2\phi - \cos 2\theta$  if  $\log \sin(\theta + i\phi) = A + iB$ . (12+8)
21. Diagonalize the matrix  $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$
22. (i) A tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  whose centre is C meets the circle  $x^2 + y^2 = a^2 + b^2$  at Q and Q'. Prove that CQ and CQ' are conjugate diameters of the ellipse.
- (ii) Derive polar equation of a conic. (10+10)