LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SECOND SEMESTER – NOVEMBER 2016

MT 2812 - PARTIAL DIFFERENTIAL EQUATIONS

Date: 08-11-2016 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer all questions. Each question carries 20 marks.

1. (a) Show that the equations $xp - yq = x$, $px^2 + q = xz$ are compatible and solve them. **(5)**

OR

(b) Eliminate the arbitrary function f from the relation $z = xy + f(x^2 + y^2)$. **(5)**

(c) Find the characteristic of the equation $pq = z$ and determine the integral surface which passes through the parabola $x = 0$, $y^2 = z$. **(15)**

OR

(d) Find the complete integral for the following equations using Jacobi's method:

(i) $p^2x + q^2y = z$ (ii) $xpq + yq^2 = 1$ (iii) $p = (z + qy)^2$. **(4 + 5 + 6)**

2. (a) If f and g are arbitrary function, show that $u = f(x - vt + i\alpha y) + g(x - vt - i\alpha y)$ is a solution of $u_{xx} + u_{yy} = \frac{1}{c^2}u_{tt}$ provided $\alpha^2 = 1 - \frac{v^2}{c^2}$. **(5)**

OR

(b) Prove that $L(u) = c^2u_{xx} - u_{tt}$ is a self adjoint. **(5)**

(c) Obtain the canonical forms of parabolic, elliptic and hyperbolic partial differential equations. **(15)**

OR

(d) Reduce the equation $u_{xx} + y^2u_{yy} = y$ to canonical form. **(15)**

3. (a) Derive Laplace equation. **(5)**

OR

(b) Obtain one-dimensional wave equation. **(5)**

(c) State and prove Interior Dirichlet problem for a circle. **(15)**

OR

(d) Determine the solution of heat conduction equation in spherical polar coordinates. **(15)**

4. (a) A uniform string of length L is stretched tightly between two fixed points at $x = 0$ and $x = l$. If it is displaced a small distance d at a point $x = b$, $0 < b < l$, and released from rest at time $t = 0$, find an expression for the displacement at subsequent times. **(5)**

OR

(b) Show that the Green's function $G(\bar{r}, \bar{r'})$ has the symmetry property. **(5)**

(c) Find the solution of the initial value problem given by $\frac{\partial^2 u}{\partial x^2} = k\frac{\partial u}{\partial t}$, $0 < x < l$, $0 < t < \infty$ subject to the conditions $u(0, t) = 0$, $u(l, t) = g(t)$, $0 < t < \infty$, $u(x, 0) = 0$, $0 < x < l$ using Laplace transform method. **(15)**

OR

(d) Obtain the solution of interior Dirichlet problem for a sphere using Green's function method. **(15)**

5. (a) Find the resolvent kernel for Kernel $K(x, t) = x - 2t$, $0 \le x \le 1, 0 \le t \le 1$. **(5)**

OR

(b) Show that all iterated kernels of a symmetric kernel are also symmetric. **(5)**

(c) Find the solution of Volterra integral equation of second kind by successive approximations. **(15)**

OR

(d) State and prove Hilbert- Schmidt theorem. **(15)**

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