



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – PHYSICS

THIRD SEMESTER – NOVEMBER 2016

MT 3100 - ALLIED MATHEMATICS FOR PHYSICS

Date: 12-11-2016
Time: 09:00-12:00

Dept. No.

Max. : 100 Marks

Part A

Answer **ALL** the questions

(10 x 2 = 20)

1. Evaluate $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$.
2. Prove that the sub tangent to the curve $y = a^x$ is of constant length.
3. Find $L[t \sin at]$.
4. Evaluate $L^{-1} \left[\frac{s}{(s+2)^2} \right]$.
5. If $\tan \frac{x}{2} = \tanh \frac{x}{2}$, prove that $\cos x \cosh x = 1$.
6. Expand $\tan 5\theta$ in terms of $\tan \theta$.
7. Prove that the matrix $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ is orthogonal.
8. If $y = e^{2x}$, prove that $y_4 - 16y = 0$.
9. Derive the expansion of $\log \frac{1+x}{1-x}$.
10. Write the general formula in Poisson's distribution.

Part B

Answer any **FIVE** questions

(5 x 8 = 40)

11. If $y = \sin^{-1} x$, prove that $(1-x^2)y_2 - xy_1 = 0$ and hence prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$
12. Find $I[t e^{-2t} \cos 3t]$.
13. If $\cos(x + iy) = \cos \theta + i \sin \theta$, then prove that $\cos 2x + \cosh 2y = 2$.
14. Sum the binomial series $\frac{5}{3.6} + \frac{5.7}{3.69} + \frac{5.79}{3.69.12} + \dots$
15. Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$.
16. If $\sin(A + iB) = x + iy$, then
Prove that (i) $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$ (ii) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$
17. Evaluate $L^{-1} \left[\frac{s}{(s+3)^2 + 4} \right]$
18. Ten coins are tossed simultaneously. Find the probability of getting at least seven heads.

Part C

Answer Any **TWO** Questions.

(2 x 20 = 40)

19. (a) Prove that $1 + \frac{2^4}{2!} + \frac{3^4}{3!} + \frac{4^4}{4!} + \dots \infty = 15e$

(b) Find the mean and standard deviation for the following table, giving the age distribution of 542 members.

Age in years	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of members	3	61	132	153	140	51	2

(10+10)

20. (a) Prove that $\sin^5 \theta = \frac{1}{16}(\sin 5\theta - 5 \sin 3\theta + 10 \sin \theta)$.

(b) Expand $\sin^3 \theta \cos^4 \theta$ in terms of sines of multiples of angles. **(10+10)**

21. (a) Find the maxima and minima of $2x^3 - 3x^2 - 36x + 10$.

(b) Find the angle at which the radius vector cuts the curve $\frac{l}{r} = 1 + e \cos \theta$. **(10+10)**

22. (a) A card is drawn from a well – shuffled pack of playing cards. What is the probability that it is either a spade or an ace?

(b) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$ **(5 +15)**
