LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2016**

MT 5407 – FORMAL LANGUAGES AND AUTOMATA

Date: 09-11-2016 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

PART-A

Answer ALL the questions: **(10 x 2=20)**

1. Define the language accepted by an NFA.
2. Draw a DFA accepting the set of all strings over {0, 1} with three consecutive zero's.
3. Prove that any finite subset is regular.
4. Define context-sensitive language.
5. Define ambiguous grammar and give an example.
6. Show that $L = \{a^p : p \text{ is a prime}\}$ is not regular.
7. Define an ϵ- free homomorphism.
8. Write a grammar for the language $L = \{a^n b^n / n \geq 1\}$.
9. Define the language star.
10. Define Greibach normal form.

PART-B

Answer any FIVE questions: **(5 x 8=40)**

11. Construct a finite automaton M accepting $\{ab, ba\}$.
12. Draw the state diagram representing the (NFA) given in the table, where M is given by $M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \delta, q_0, \{q_3\})$

δ	0	1
q_0	$\{q_0, q_1\}$	$\{q_0, q_2\}$
q_1	$\{q_3\}$	ϕ
q_2	ϕ	$\{q_3\}$
q_3	$\{q_3\}$	$\{q_3\}$

13. Prove that union of two regular set is regular.
14. Let $G = \{N, T, P, S\}$ $N = \{S, B\}$ and $T = \{a, b, c\}$. P consists of the following productions:

(i) $S \rightarrow aSB$ (iii) $bB \rightarrow bbc$
(ii) $S \rightarrow abc$ (iv) $cB \rightarrow Bc$

Then show that $L(G) = \{a^n b^n c^n / n \geq 1\}$ is a CSL.

15. Let $G=\{N,T,P,S\}$, where $N=\{S,A\}$ $T=\{a,b\}$ and P consists of the rules

1. $S\to aAb$ 2. $S\to abSb$ 3. $S\to a$ 4. $A\to bS$ 5. $A\to aAAb$

Find the leftmost and rightmost derivations for the string *abab* .

16. Prove that the families of *PSL*, *CSL*, *CFL* and *RL* are closed under union.

17. Consider the grammar $G=\{N,T,P,S\}$ where

$$N=\{S,(P_r),(VP),V,(NP),A,N,(Aux),P\},\ T=\{They, are, flying, planes\},$$

$$P=\left\{\begin{array}{l} S\to (P_r)(VP), P_r\to They, VP\to (V)(NP), V\to are, NP\to (A)(N), \\ A\to flying, N\to planes, V\to (Aux)(P), Aux\to are, NP\to N, P\to flying \end{array}\right\},$$ and S

is the start symbol, generates the language consisting of the single sentence,

$\{They\ are\ flying\ planes\}$.

18. Prove that $L(G)=\{a^n b^n c^n / n\geq 1\}$ is not a Context Free Language (CFL).

PART - C

Answer any TWO questions: **(2 x 20=40)**

19. Let $M=(\{q_0,q_1,q_2,q_3\},\{a,b\},\delta,q_0,\{q_1\})$ is a finite automaton δ is given by

$\delta(q_0,a)=q_1$, $\delta(q_1,a)=q_3$ $\delta(q_2,a)=q_2$, $\delta(q_3,a)=q_2$, $\delta(q_0,b)=q_2$, $\delta(q_1,b)=q_0$, $\delta(q_2,b)=q_2$, $\delta(q_3,b)=q_2$

(a) Represent M by its state table.

(b) Represent M by its state diagram.

(c) Which of the following strings are accepted by M ?

(i) *ababa* (ii) *aabba* (iii) *aaaab* (iv) *bbbaa*

(6+6+8)

20. (i) State and prove the pumping lemma.

(ii) Construct a deterministic finite automaton (FA) equivalent to a given NFA where,

$M=(\{q_0,q_1,q_2,q_3\},\{0,1\},\delta,q_0,\{q_3\})$, δ is given in the following table:

δ	a	b
q_0	$\{q_0,q_1\}$	$\{q_0\}$
q_1	ϕ	$\{q_2\}$
q_2	ϕ	ϕ

(8+12)

21. (i) Let $G = (\{S, Z, A, B\}, \{a, b\}, P, S)$ where P consists of the following productions:

1. $S \rightarrow aSA$	2. $S \rightarrow aZA$	3. $Z \rightarrow bZB$
4. $Z \rightarrow bB$	5. $BA \rightarrow AB$	6. $AB \rightarrow Ab$
7. $bB \rightarrow bb$	8. $bA \rightarrow ba$	9. $aA \rightarrow aa$

Then show that $L(G)=\{a^n b^m a^n b^m / n,m \geq 1\}$.

(ii) Prove that the family of CFL is closed under substitution.

(12+8)

22. (i) State and prove Chomsky Normal form.

(ii) Let $L = \{a^n b^n / n \geq 1\}$, then $G = \{N, T, P, S\}$ where $N = \{S\}$, $T = \{a, b\}$ and $P = \{S \rightarrow aSb, S \rightarrow ab\}$, verify Chomsky normal form and generates context-free language.

(10+10)

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