LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – NOVEMBER 2016

MT 5408 - GRAPH THEORY

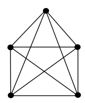
Date: 11-11-2016 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

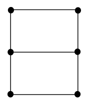
PART – A

Answer ALL questions **(10 x 2 = 20)**

1. Define a bipartite graph with example.
2. Define removal of the vertices with example.
3. When a $v_n - v_0$ walk is said to be closed?
4. Define distance between any two vertices of a graph.
5. Define an Eulerian graph and given an example.
6. Prove that every Hamiltonian graph is 2-connected.
7. Define a spanning tree with examples.
8. Define an eccentricity of a vertex v in a connected graph G.
9. Show that $K_{3,3}$ is not planar.
10. Find the chromatic number for the following graphs.



(a)



(b)

PART – B

Answer any FIVE questions **(5 x 8 = 40)**

11. (a) Prove that any graph G the number of vertices of odd degree is even.

    (b) Prove that $\Gamma(G) = \Gamma(\bar{G})$. **(4 + 4)**

12. If Let G_1 be a (p_1, q_1) graph and G_2 be a (p_2, q_2) graph then prove that

    (i) $G_1 + G_2$ is a $(p_1 + p_2, q_1 + q_2 + p_1p_2)$ graph.

    (ii) $G_1 \times G_2$ is a $(p_1p_2, q_1p_2 + q_2p_1)$ graph.

13. Define connected graph and prove that a graph G with P vertices and $\delta \geq \frac{p-1}{2}$ is connected.

14. (a) Define cut vertex with examples.

    (b) Prove that every non - trivial connected graph has atleast two vertices which are not cut vertex. **(3 + 5)**

15. If G is a graph with $p \geq 3$ vertices and $\delta \geq p/2$, then prove that G is Hamiltonian.

16. Prove that every planar graph is 5-colourable.

17. State and prove Euler's theorem.

18. (a) If G is a (p , q) plane graph with r faces and k components then prove that $p - q + r = k + 1$.

    (b) If G is a connected (p , q) planar graph with no triangle and $p \geq 3$, then prove that $q \leq 2p - 4$.

PART – C

Answer any TWO question **(2 x 20 = 40)**

19. (a) Show that in a group of two or more people there are always two with exactly same number of friends inside the room

(b)The maximum number of edges among all *p* vertex graphs with no triangles is $\frac{p^2}{4}$. **(6 + 14)**

20. (a) Prove that any self complementary graph has $4n$ or $4n+1$ vertices.

(b) Prove that a graph *G* with atleast two vertices is bipartite iff all its cycle are of even length.

(5 +15)

21. (a) Prove that the following statements are equivalent for a connected graph *G*

(i) *G* is eulerian.

(ii) Every point of *G* has even degree.

(iii)The set of edges of *G* can be partitioned into cycles.

(b) If *G* is a graph in which the degree of every vertex is atleast two then prove that *G* is contains a cycle. **(12 + 8)**

22. (a) Let G be a (p, q) graph then prove that the following statements are equivalent

(i) *G* is a tree.

(ii) Every two points of G are joined by a unique path.

(iii) *G* is connected and $p = q + 1$.

(iv) *G* is acyclic and $p = q + 1$.

(b) Prove that every non – trivial tree has atleast two vertices of degree one. **(12 + 8)**

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