LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2017**

16PMT3ES02– DIFFERENTIAL GEOMETRY

Date: 10-11-2017 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Answer all the questions

1. (a) Show that the ratio of the arc and the chord connecting two points P and Q on a curve approaches unity when Q approaches P. (5)

(OR)

(b) Obtain the equation of the tangent at a point on the curve of intersection of two surfaces $f_1(x,y,z)=0$ and $f_2(x,y,z)=0$. (5)

(c) Derive the equation of the osculating plane at the point on the space curve and hence find the equation of the osculating plane for the vector $\vec{r}=(u,u^2,\ u^3)$. (15)

(OR)

(d) (i) Show that the tangent at the point of the curve of intersection of the ellipsoid and the confocal conic with parameter λ is given by $\frac{x(X-x)}{a^2(b^2-c^2)(a^2-\lambda)}=\frac{y(Y-y)}{b^2(c^2-a^2)(b^2-\lambda)}=\frac{z(Z-z)}{c^2(a^2-b^2)(c^2-\lambda)}$.

(ii) Find the length of the circular helix $\vec{r}=acosu\,\vec{\imath}+asinu\,\vec{\jmath}+bu\,\vec{k}$, $-\infty<u<\infty$ varies from the point $(a,0,0)$ to $(a,0,2\pi b)$. Also obtain the equation in terms of parameter s. (10+5)

2. (a) Show that if the circle $lx+my+nz=0$, $x^2+y^2+z^2=2cz$ has three point of contact at the origin with the paraboloid $ax^2+by^2=2z$ then $c=\frac{l^2+m^2}{bl^2+am^2}$.

(OR)

(b) Derive the equation of an involute of a space curve. (5)

(c) State and prove fundamental theorem of space curves. (15)

(OR)

(d) Derive the Riccati equation from the general solution to the natural equations of a space curve. (15)

3. (a) Define envelope, developable surface, essential singularity and artificial singularity. (5)

(OR)

(b) Find the angle between two curves lying on a surface at a point of intersection of two curves. (5)

(c) Explain the first fundamental form of a surface and give its geometrical interpretation. (15)

(OR)

(d) Derive the equation of polar and tangential developables associated with a surface. (15)

4. (a) State and prove Meusnier's theorem. (5)

(OR)

(b) Find the principal curvature and principal direction at any point on a surface
$x=a(u+v), y=a(u-v), z=uv$. (5)

(c) (i) Find the first fundamental form and the second fundamental form of the curve $x = a\ cos\theta\ sin\varphi$, $y = a\ sin\theta sin\ \varphi,\ z = a\ cos\ \varphi$.

(ii) With usual notations, prove that the necessary and sufficient condition that the lines of curvature may be a parametric curve is that $f = 0$ and $F = 0$. (10+5)

(OR)

(d) (i) Derive the equation satisfying principal curvature at a point on a surface.
(ii) State and prove Euler's theorem. (7+8)

5. (a) Derive Weingarton equation. (5)

(OR)

(b) Prove that in a region R of a surface of constant positive Gaussian curvature without umbilics, the principal curvature take their extreme values at the boundary. (5)

(c) Derive Gauss equation in terms of Christoffel's symbol. (15)

(OR)

(d) State the fundamental theorem of Surface Theory and demonstrate it in the case of unit sphere. (15)

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