LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – NOVEMBER 2017

16PMT3MC01/MT3810 - TOPOLOGY

Date:01-II-2017 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Answer all the questions.

I. a)1) Let X be a metric space. Prove that arbitrary union of open sets is open.

OR

a)2) Let X be a metric space. Prove that any finite intersection of open sets is open. (3)

b)1) State and prove Cantor's intersection theorem.
b)2) Let X and Y be metric spaces and f a mapping of X into Y. Then prove that f is continuous $\Leftrightarrow$ $f^{-1}(G)$ is open whenever G is open in Y. (9+8)

OR

c)1) Construct Cantor's set.
c)2)) Let X be a metric space and let Y be a complete metric space, and let A be a dense subspace of X. If f is uniformly continuous mapping of A into Y then prove that f can be extended uniquely to a uniformly continuous mapping g of X into Y. (5+12)

II. a)1) Define a topological space and give an example.

OR

a)2) Define a metrizable space. Does every topological space become a metrizable space? Under what condition does every topological space become metrizable? (3)

b)1) State and prove Lindelof's theorem.
b)2) State and prove Lebesgue's covering lemma. (7+10)

OR

c) Prove that a topological space is compact if every subbasic open cover has a finite subcover. (17)

III. a)1) Prove that every compact subspace of a Hausdorff space is closed.

OR

a)2) Prove that a one to one continuous mapping of a compact space onto a Hausdorff space is a homeomorphism. (3)

b) State and prove Tietze extension theorem. (17)

OR

c) State and prove Urysohn Embedding theorem. (17)

IV. a)1) Prove that a topological space X is disconnected iff there exists a continuous mapping of X onto the discrete two-point space {0,1}.

OR

a)2) Prove that any continuous image of a connected space is connected. (3)

b)1) Prove that the subspace of a real line is connected iff it is an interval also prove that $\mathbb{R}$ is connected.

b)2) Prove that the spaces $\mathbb{R}^n$ and $\mathbb{C}^n$ are connected. (9+8)

OR

c)1) Prove that the product of any non-empty class of connected space s is connected.

c)2) Let X be a compact Hausdorff space. Prove that X is totally disconnected iff it has an open base whose sets are also closed. (7+10)

V. a)1) State complex Stone-Weierstarss theorem.

OR

a)2) Prove that X_∞ is Hausdorff. (3)

b)State and prove Weierstrass approximation theorem. (17)

OR

c) State and prove Real Stone Weierstrass theorem. (17)

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