LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION –**MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2017**

MT 1502 – ALGEBRA AND CALCULUS - I

Date: 14-11-2017 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART- A

Answer ALL Questions (10 × 2 = 20)

1. Find the nth derivative of $\sin x$.

2. Find the polar subtangent of the curve $r = e^{\theta cot\alpha}$.

3. Write down the conditions for maxima and minima of functions of two variables.

4. Define curvature of a curve at a point on the curve..

5. Find the radius of curvature at (1, ½) on the curve $2y = x(1 - x + x^2)$..

6. Define evolute and involute of a curve.

7. Form a rational cubic equation which shall have roots 1, $3 - \sqrt{-2}$.

8. Write the condition for the roots of the equation to be in harmonic progression.

9. Show that the equation $x^7 - 3x^4 + 2x^3 - 1 = 0$ has at least four imaginary roots.

10. If α, β, γ are the roots of the equation $x^3 + 6x^2 + 11x + 6 = 0$ form the equation whose roots are $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$.

PART- B

Answer any FIVE questions(5× 8 = 40)

11. If $y = e^{a \sin^{-1} x}$, prove that $(1 - x^2)y_2 - xy_1 = 0$. Hence show that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$$

12.Find the maximum and minimum values of the function $f(x, y) = x^2y^2 - x^2 - y^2$.

13. Prove that the radius of curvature at any point of the cycloid $x = a(\theta + sin\theta)$ and $y = a(1 - cos\theta)$is$4acos\frac{\theta}{2}$.

14. Find the radius of curvature at the point t in the parabola $y^2 = 4ax$ and deduce the equation of the evolute.

15. Solve $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$, given that one root is -1+i.

16.If the roots of $x^3 + px^2 + qx + r = 0$ are in A.P, show that $2p^3 - 9pq + 27r = 0$.

17. Discuss the nature of the roots of $x^4 + 4x^3 - 2x^2 - 12x + a = 0$ for all real values of a.

18. Find the asymptotes of the cubic equation $y^3 - 6xy^2 + 11x^2y - 6x^3 + x + y = 0$.

PART- C
Answer any TWO questions (2× 20 = 40)

19 (a) State and prove Leibnitz formula for the nth derivative of a product of two functions.

(b) Find the angle of intersection of the cardioids $r = a(1 + cos\theta)$ and $r = b(1 - cos\theta)$

20 (a) If $u = a^3x^2 + b^3y^{\ 2} + c^3z^2$where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, find the minimum value of u.

(b) From the polar equation of the parabola show that $\rho^2 = ar$.

21. (a) Show that the sum of the eleventh powers of the roots of $x^7 + 5x^4 + 1$=0 is zero.

(b) Solve the reciprocal equation $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$.

22. Using Horner's method find the root of the equation $x^3 - 3x + 1 = 0$ which lies between 1 and 2 correct to two decimal places.

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