LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc.DEGREE EXAMINATION – **MATHEMATICS**

FIRSTSEMESTER – NOVEMBER 2017

MT 1817- ORDINARY DIFFERENTIAL EQUATIONS

Date: 08-11-2017 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer all questions. Each question carries 20 marks.

1. (a) Is $c_1t + c_2t^2 + c_3t^3$, $t \geq 0$ a solution of $t^3x''' - 3t^2x'' + 6tx' - 6x = 0$? (5)

(OR)

(b) State and prove Abel's formula. (5)

(c) Explain the method of variation of parameters. (15)

(OR)

(d) Discuss the various solutions of the second order linear homogenous equation with constant coefficients. (15)

2. (a) State and prove Rodrigue's Formula. (5)

(OR)

(b) With usual notation, prove that (i) $P_l(1) = 1$, (ii) $P_l'(1) = l(l+1)/2$. (5)

(c) Solve by Frobenius method, $x(1-x)\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} - y = 0$. (15)

(OR)

(d) Derive the orthogonality properties of the Legendre's polynomial. (15)

3. (a) Derive the generating function for Bessel's function. (5)

(OR)

(b) When *n* is a non-zero integer, show that $J_{-n}(x) = (-1)^n J_n(x)$. (5)

(c) State and prove the integral representation of Bessel's function. (15)

(OR)

(d) Derive the recurrence relations for Bessel's function. (15)

4. (a) Using the method of successive approximations, solve the initial value problem $x'(t) = -x(t), x(0) = 1,\ t \geq 0$. (5)

(OR)

(b) For distinct parameters λ and μ, let x and y be the corresponding solutions of the Sturm-Liouville problem such that $[pW(x,y)]_A^B = 0$. Prove that $\int_A^B r(s)x(s)y(s)ds = 0$. (5)

(c) State and prove Picard's theorem for initial value problem. (15)

(OR)

(d) Let $G(t,s)$ be the Green's function. Prove that $x(t)$ is a solution of $L(x(t)) + f(t) = 0, a \leq t \leq b$ if and only if $x(t) = \int_a^b G(t,s)f(s)\,ds$. (15)

5. (a) Explain asymptotically stable solution. (5)

(OR)

(b) Prove that the null solution of equation $x' = A(t)x$is stable if and only if a positive constant *k* exists such that $|\phi(t)| \leq k, t \geq t_0$. (5)

(c) Discuss the stability of the system $x' = Ax$by Lyapunov's method. (15)

(OR)

(d) State and prove the fundamental theorems on the stability of non-autonomous systems. (15)

****