LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc.DEGREE EXAMINATION –**MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2017**

MT 5508 / MT 5502 – LINEAR ALGEBRA

Date: 08-11-2017 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

PART A

ANSWERALL THE QUESTIONS $(\mathbf{10 * 2 = 20}\ \boldsymbol{marks})$

1. Define a vector space over a field F
2. Let V is a vector space over a field F. Prove that $\{v_1, v_2, \dots, v_m\}$ is linearly dependent set of vectors if at least one of them is the zero vector.
3. Prove that the vectors (1, 0, 0), (1,1,0) and (1,1,1) form a basis ofR^3, where R is the field of real numbers.
4. Define rank and nullity of a vector space homomorphism $T: U \to V$.
5. Normalize$(1 + 2i, 2 - i, 1 - i)$ in C^3 relative to the standard inner product.
6. Let $T \in A(v)$ and $\lambda \in F$. If λ is an eigenvalue of T, prove that $\lambda I - T$ is singular.
7. Define Nilpotent and Idempotent matrices.
8. Show that $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.
9. If $T \in A(V)$ is Hermitian, then prove that all its eigen values are real.
10. Define unitary linear transformation.

PART B

ANSWERANY FIVE QUESTIONS $(\mathbf{5 * 8 = 40}\ \boldsymbol{marks})$

11. If V is a vector space over F then show that

    i) $a0 = 0$ for $a \in F$

    ii) $(-a)v = a(-v) = -(av)$ for $a \in F, v \in V$.

    iii) If $v \neq 0$,then $av = 0$ implies that $a = 0$.
12. Prove that the union of two subspaces of a vector spaces V over F is a subspace ofV if and only if one is contained in the other
13. If V is a vector space of dimension n, then prove that

    i) Any $n + 1$ vectors in V are linearly dependent

    ii) Any set of$n$ linearly independent vectors of V is basis of V.
14. If V and W are two n-dimensional vector spaces over F, then prove that any isomorphism $T$of V onto W maps a basis of V onto a basis of W.

15. For any two vectors $u, v \in V$, prove that $\|u+v\| \leq \|u\| + \|v\|$.

16. If $\lambda \in F$ is an eigenvalue of $T \in A(V)$, then prove that for any polynomial $f(x) \in F[x]$, $f(\lambda)$ is an eigenvalue of $f(T)$.

17. Show that any square matrix A can be expressed uniquely as the sum of a symmetric matrix and a skew-symmetric matrix.

18. If $T \in A(V)$ is skew-Hermitian, prove that all of its eigenvalues are pure imaginaries.

PART C

ANSWERANY TWO QUESTIONS $(\mathbf{2 * 20 = 40}\ \boldsymbol{marks})$

19. a)If S and T are subsets of a vector space V over F,then prove that

i) $S$is a subspace of$V$ if and only if $L(S) = S$.

ii) $S \subseteq T$ implies that $L(S) \subseteq L(T)$.

iii) $L(L(S)) = L(S)$.

iv) $L(S \cup T) = L(S) + L(T)$.

b) If V is a vector space of finite dimension and W is a subspace of$V$,then prove that

$\dim V/W = \dim V - \dim W$. **(10+10)**

20. a) If $T: U \rightarrow V$ is a homomorphism of two vector spaces over F and U has finite dimension then prove that $dim\ U = dim\ ker\ T + dim\ Im\ T$

b) If U and V are vector spaces over F, and if T is a homomorphism of U onto V with kernel W, then prove that $U/W \cong V$. **(10+10)**

21. Apply the Gram-Schmidt orthonormalization process to obtain an orthonormal basis for the subpace of R^4 generated by the vectors (1,1,0,1), (1,-2,0,0) and (1,0,-1,2).

22. a) Let $V = R^3$, and let $T \in A(V)$ be defined by $T(a_1, a_2, a_3) = (3a_1 + a_3, -2a_1 + a_2, -a_1 + 2a_2 + 4a_3)$. What is the matrix of T relative to the basis $v_1 = (1,0,1), v_2 = (-1,2,1), v_3 = (2,1,1)$?

b) Investigate for what values of λ, μ the system of equations

$x_1 + x_2 + x_3 = 6, x_1 + 2x_2 + 3x_3 = 10, x_1 + 2x_2 + \lambda x_3 = \mu$ over the rational field has

i) no solution ii) a unique solution iii) an infinite number of solutions. **(10+10)**

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