LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2018**

16/17PMT3ES02 – DIFFERENTIAL GEOMETRY

Date: 02-11-2018 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

Answer all the questions

1. (a) Prove that the curvature is the rate of change of the angle of contingency with respect to the arc length. (5)

(OR)

(b) Obtain the equation of the tangent at a point on the curve of intersection of two surfaces $f_1(x,y,z)=0$ and $f_2(x,y,z)=0$. (5)

(c) Define an osculating plane and derive the equation of the osculating plane at the point on the space curve. (15)

(OR)

(d) (i) Show that the tangent at the point of the curve of intersection of the ellipsoid and the confocal conic with parameter λ is given by $\frac{x(X-x)}{a^2(b^2-c^2)(a^2-\lambda)}=\frac{y(Y-y)}{b^2(c^2-a^2)(b^2-\lambda)}=\frac{z(Z-z)}{c^2(a^2-b^2)(c^2-\lambda)}$.

(ii) Show that the ratio of the arc and the chord connecting two points P and Q on a curve approaches unity when Q approaches P. (10+5)

2. (a) Find the plane that has three point of contact at origin with the curve $x=u^4-1$, $y=u^3-1$, $z=u^2-1$. (5)

(OR)

(b) Prove that the necessary and sufficient condition that a curve be of constant slope is that the ratio of curvature to the torsion is a constant. (5)

(c) State and prove fundamental theorem of space curves. (15)

(OR)

(d) If the general equation of Riccati equation $\frac{df}{ds}=\frac{-i\tau}{2}-ikf+\frac{i\tau}{2}f^2$ is found in the form $f=\frac{cf_1+f_2}{cf_3+f_4}$ where f_1,f_2,f_3,f_4 are functions of s then prove that the curve is given by the equation $x=\int^s \alpha_1 ds$, $y=\int^s \alpha_2 ds$, $z=\int^s \alpha_3 ds$ where $\alpha_1=\frac{{f_1}^2-{f_2}^2-{f_3}^2+{f_4}^2}{2(f_1f_4-f_2f_3)}$,
$\alpha_2=\frac{i({f_1}^2+{f_2}^2-{f_3}^2-{f_4}^2)}{2(f_1f_4-f_2f_3)}$, $\alpha_3=\frac{f_3f_4-f_2f_1}{f_1f_4-f_2f_3}$ has $k(s)$ and $\tau(s)$ as curvature and torsion. (15)

3. (a) What are the types of singularities? Explain briefly. (5)

(OR)

(b) Find the angle between two curves lying on a surface at a point of intersection of two curves. (5)

(c) Explain the first fundamental form of a surface and give its geometrical interpretation. (15)

(OR)

(d) Derive the equation of polar and tangential developables associated with a surface. (15)

4. (a) With usual notations, prove that the necessary and sufficient condition that the lines of curvature may be a parametric curve is that $f = 0$ and $F = 0$. (5)

(OR)

(b) Find the principal curvature and principal direction at any point on a surface $x = a(u + v), y = a(u - v), z = uv$. (5)

(c) (i) Find the first fundamental form and the second fundamental form of the curve $x = a\, cos\theta\, sin\varphi$, $y = a\, sin\theta sin\, \varphi,\ z = a\, cos\, \varphi$.

(ii) State and prove Meusnier's theorem. (10+5)

(OR)

(d) Derive the equation satisfying principal curvature at a point on a surface and the equation of principal direction at a point. (15)

5. (a) Derive Weingarton equation. (5)

(OR)

(b) Derive the Christoffel's symbol of second kind. (5)

(c) Derive Mainardi Codazzi equation. (15)

(OR)

(d) State the fundamental theorem of Surface Theory and demonstrate it in the case of unit sphere. (15)

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