LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2018**

MT 1502– ALGEBRA AND CALCULUS - I

Date: 24-10-2018 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

PART – A

Answer all questions: **(10 X 2 = 20)**

1. Find the n^{th} derivative of $y = \sin(2x+3)$.
2. Prove that the sub tangent to the curve $y = a^x$ is of constant length.
3. Write the condition for the maxima and minima of two variables.
4. Write the steps used in Lagrange's method of undetermined multiples.
5. Find the radius of curvature of the curve $y = e^x$ at the point where it crosses the y-axis.
6. Define evolute of a curve.
7. Determine the quadratic equation having $1-\sqrt{5}$ as a root.
8. Find $\alpha\beta + \beta\gamma + \gamma\alpha$ of the equation $81x^3 - 18x^2 - 36x + 8 = 0$.
9. State Descarte's rule of signs for positive roots.
10. If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, find the value of $\sum \frac{1}{\beta + \gamma - \alpha}$.

PART – B

Answer any five questions: **(5 X 8 = 40)**

11. If $y = a\cos(\log x) + b\sin(\log x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.
12. Find the angle of intersection of the cardioids $r = a(1 + cos\theta)$ and $r = a(1 - cos\theta)$.
13. Find the maximum or minimum values of $x^3 y^2 (6 - x - y)$.
14. Find the radius of curvature at 't' of the curve $y^2 = 4ax$.
15. Prove that the $(p-r)$ equation of the cardioids $r = a(1-\cos\theta)$ is $p^2 = \frac{r^3}{2a}$.
16. Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that two of its roots are equal in magnitude and opposite in sign.

17. Find the sum of fifth powers of the roots of $x^4 - 3x^3 + 5x^2 - 12x + 4 = 0$.

18. Show that the equation $x^7 - 3x^4 + 2x^3 - 1 = 0$ has at least four imaginary roots.

PART – C

Answer any two questions: **(2 X 20 = 40)**

19. (i)If $y^{1/m} + y^{-1/m} = 2x$, prove that $(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$

(ii) Find the maximum or minimum values of $2(x^2 - y^2) - x^4 + y^4$. **(10+10)**

20. Show that the evolute of the cycloid $x = a(\theta - \sin\theta); y = a(1 - \cos\theta)$ is another cycloid.

21. (i) Solve the equation $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$ of which one root is $-1 + \sqrt{-1}$.

(ii) Solve the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$. **(10+10)**

22. (i) Using Newton's method of divisors, solve the equation $x^4 - 2x^3 - 13x^2 + 38x - 24 = 0$.

(ii) Solve the equation $x^3 - 9x^2 + 108 = 0$, using Cardon's method. **(10+10)**

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