LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIFTH SEMESTER – NOVEMBER 2018

MT 5406 – COMBINATORICS

Date: 01-11-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

SECTION-A

Answer all the questions. **(10×2=20)**

1. How many 3-digit numbers can be formed by using the 6 digits 2, 3, 4, 5, 6 and 8, if a) repetitions are allowed? b) repetitions are not allowed?
2. Define the falling factorial polynomial.
3. 10 teams participate in a tournament. The first team is awarded a gold medal, the second a silver model and the third a bronze medal. In how many ways can the medals be distributed?
4. State the multinomial theorem.
5. Define the Stirling number of the second kind.
6. Find the sequences of the ordinary generating functions $3x^2 + e^{2x}$ and $(3 + x)^3$.
7. Find the coefficient of x^{27} in $(x^4 + x^5 + x^6 + \cdots)^5$ and in $(x^4 + 2x^5 + 3x^6 + \cdots)^5$.
8. Write the exponential generating functions of the following sequences:
   a) $\{1,2,3,4, \dots\}$
   b) $\{0,0,0,1,1,1, \dots\}$
9. Evaluate $\phi(3528)$
10. Calculate $per\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

SECTION-B

Answer any five questions. **(5×8=40)**

11. Define Stirling number of first kind and derive recurrence formula for the Stirling number of the first kind.
12. Define the rising factorial $[m]^n$ and prove that the number of distribution of n distinct objects into m distinct boxes with the objects in each box arranged in a definite order is the rising factorial.
13. Prove the following:
    i) $D_n = n!\left[\frac{1}{2!} - \frac{1}{3!} + \cdots \frac{(-1)^n}{n!}\right]$.
    ii) 97 is the twenty-fifth prime. (4+4)
14. Derive the formula to find the sum of first n natural numbers using its recurrence formula given by $a_n - a_{n-1} = n, n \geq 1$.
15. a) In an experiment, 4 differently colored dice are thrown simultaneously and the numbers are added. Find the i) number of distinct experiments such that the total is 18 and ii) the total is 18 and green die shows an even number.
    b) Find the number of ways of forming a committee of 9 people drawn from 3 different parties so that no party has an absolute majority in the party. (5+3)
16. Prove the multinomial theorem and hence find the coefficient of $x_1^2 x_3 x_4^3 x_5^4$ in the expression $(x_1 + x_2 + x_3 + x_4 + x_5)^{10}$.
17. State and prove Sieve's formula and hence find the number of positive integers less than 601 that are not divisible by 3 or 5 or 7.

18. State and prove Burnside Frobenius theorem.

SECTION-C

Answer any two questions. **(2×20=40)**

19. a) Prove that the element $f = \sum_{k=0}^{\infty} \alpha_k t^k \in \mathbb{R}[t]$ has an inverse in $\mathbb{R}[t]$ if and only if α_0 has an inverse in $\mathbb{R}$.

b) A box contains many identical red, blue, white and green marbles. Find the ordinary generating function corresponding to the problem of finding the number of ways of choosing r marbles from the box such that the sample does not have more than 2 red, more than 3 blue, more than 4 white and more than 5 green and hence find the number of ways of choosing 10 marbles from the box containing the mentioned sample. (12+8)

20. State and prove the Ménage problem.

21. a) Find the rook polynomial for a 2×2 Chess board by the use of expansion formula.

b) An executive attending a week long seminar has 5 suits of different colors. On Mondays she does not wear blue or green, on Tuesdays she does not wear red or green, on Wednesdays she does not wear blue or white or yellow, on Fridays, she does not wear white. How many ways can she does without repeating a color for the seminar. (8+12)

22. Find all elements of the group G of symmetries of a square. Count the distinguishable colorings of the 4 vertices, if each vertex is to be either red or blue. Exhibit in a diagram the patterns. Also find the Pattern Inventory of G.

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