LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2019**

16/17UMT5MC01 – REAL ANALYSIS

Date: 29-10-2019 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

PART A

Answer ALL the questions. **(10X 2=20)**

1. Define order-complete.
2. If $x, y \in R$ then prove that $|x+y| \leq |x| + |y|$.
3. Define Metric Space.
4. Define Accumulation point.
5. Define complete metric space.
6. Define uniformly continuous.
7. If *f* is differentiable at *c* then prove that *f* is continuous at *c*.
8. State Generalized mean value theorem.
9. Define Monotonic functions.
10. Define Total Variation.

PART B

Answer any FIVE questions. **(5X 8=40)**

11. Prove that $e = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots$ is irrational.
12. Prove that R is uncountable.
13. If f, g are continuous at $x_0 \in X$, then prove that (i) fg is continuous at $x_0 \in X$ and (ii) $\frac{1}{f}$ is continuous at $x_0 \in X$.
14. Prove that Euclidean space R^k is complete.
15. State and prove Rolle's theorem.
16. Let *f* be of bounded variation on $[a,b]$ and $x \in [a, b]$. Define $V: [a, b] \to R$ as follows:
$V(a) = 0$, $V(x) = V_f[a, x], a < x \leq b$.
Then prove that functions V and $V - f$ are both increasing functions on $[a, b]$.
17. State and prove Intermediate value theorem for derivatives.
18. State and prove Heine Borel theorem.

PART C

Answer any TWO questions. **(2 X20=40)**

19. (a) State and prove Cauchy- Schwarz inequality

(b) Prove that every subset of a countable set is countable.

(10 +10)

20.(a) Let M = $R^n. If\ x = (x_1, x_2, .. x_n)$ and $y = (y_1, y_2, ... y_n)$ are points in R^n, define

$d(x, y) = [\sum_{k=1}^{n}(x_k - y_k)^2]^{\frac{1}{2}}$. Then show that (M, d) is a metric space.

(b) State and prove Bolzano-Weierstrass theorem for R.

(10 +10)

21. (a) Prove that every convergent sequence is Cauchy sequence.

(b) State and prove Taylor's theorem.

(10 +10)

22. (a) Let *f* be functions of bounded variation defined on $[a, b]$ and $c \in (a, b)$. Prove that *f* is of bounded variation on $[a, c]$ as well as on $[c, b]$ and $V_f[a, b] = V_f[a, c] + V_f[c, b]$.

(b) Let (X, d_1) and (Y, d_2) be metric spaces .Then prove that a map $f: X \to Y$ is continuous on X if and only if $f^{-1}(G)$ is open in X for every open set G in Y.

(10 +10)