LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2019**

PMT 1505 – PROBABILITY THEORY AND STOCHASTIC PROCESS

Date: 09-11-2019 Dept. No. Max. : 100 Marks
Time: 01:00-04:00

Answer **ALL** questions:

1. (a) Prove that if X and Y are independent then $cov(X,Y)=0$. **(5)**

(OR)

(b) Derive the cumulant generating function of χ^2 − distribution. **(5)**

(c) A random variable X has the following probability distribution function

values of x	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	[illegible]	2[illegible]	7[illegible]k

(i) Determine the value of k.
(ii) Find $P(X<6)$, $P(X\geq 6)$ and $P(0<X<5)$.
(iii) What is the minimum value of a, for which $P(x\leq a)\geq 0.5$. **(15)**

(OR)

(d) Two random variables x and y have joint density functions

$$f(x,y)=\begin{cases} x+y, & 0\leq x\leq 1, 0\leq y\leq 1 \\ 0, & otherwise \end{cases}$$

Find the correlation coefficient of X and Y. **(15)**

2. (a) State and prove Chebychev's inequality. **(5)**

(OR)

(b) If $E(X^2)=\sum k^2 p_k$ exists, then prove that $E(X^2)=P''(1)+P'(1)=2Q'(1)+Q(1)$ and $V(X)=2Q'(1)+Q(1)-\{Q(1)\}^2=P''(1)+P'(1)-\{P'(1)\}^2$. **(5)**

(c) State and prove the necessary and sufficient condition for the weak law of large numbers. **(15)**

(OR)

(d) State and prove De-Moivre's Laplace theorem. **(15)**

3. (a)Write short note on the characteristics of estimators. **(5)**

(OR)

(b) Prove that the maximum likelihood estimate of the parameter α of a population having density function $\frac{2}{\alpha^2}(\alpha-x), 0<x<\alpha$, for a sample of unit size is $2x$, x being the sample value. Show also that the estimate is biased. **(5)**

(c) (i) A random sample $(X_1, X_2, X_3, X_4, X_5)$ of size 5 is drawn from a population with unknown mean μ. Consider the following estimators to estimate μ:

(i) $t_1 = \frac{X_1+X_2+X_3+X_4+X_5}{5}$ (ii) $t_2 = \frac{X_1+X_2}{2} + X_3$ (iii) $t_2 = \frac{2X_1+X_2+\alpha X_3}{3}$, where α is such that t_2 is an unbiased estimator of μ. Find α. Are t_1 and t_2 unbiased? State giving reasons, the estimator which is best among t_1, t_2 and t_3.

(ii) If T_n is a consistent estimator of $\gamma(\theta)$ and $\psi(\gamma(\theta))$ is a continuous function of $\gamma(\theta)$, then prove that $\psi(T_n)$ is a consistent estimator of $\psi(\gamma(\theta)$. **(10+5)**

(OR)

(d) State and prove the sufficient conditions for Consistent Estimators **(15)**

4. (a) Brief the following:
(i) Type I and Type II error
(ii) Probability form of Type I and Type II error **(2+3)**

(OR)

(b) In one sample of eight observation, the sum of squares of deviation of the sample values from the sample mean is 84.4 and in the other sample of 10 observations it was 102.6. Test the difference of significance at 5% level. For F-distribution the given degrees of freedom for (7,9) is 3.29.

(5)

(c) Let P be the probability that a coin will fall head in the single task in order to test $H_0: p = \frac{1}{2}$, $H_1: p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I and II errors and power of the test.

(OR)

(d) State and prove Neyman-Pearson Lemma. **(15)**

5. (a) Write short note on Markov process. **(5)**

(OR)

(b) Explain the four different classes of stochastic processes. **(5)**

(c) Let $\boldsymbol{P}$ be the transition probability matrix of a homogeneous finite Markov chain with elements $p_{ij}(i, j = 0,1,2, \ldots, k-1)$. Then prove that the n-step transition probabilities $p_{ij}^{(n)}$ are obtained as the elements of the matrix $\boldsymbol{P}^n$. **(15)**

(d) (i) If the initial vector $\boldsymbol{P}^{(0)}$ is given, then prove that the n −step transition probabilities are $\boldsymbol{P}^{(n)} = \boldsymbol{P}^{(0)}\boldsymbol{P}^{n}$, $n = 1,2, \ldots$.

(ii) Explain Markov Chain with examples. **(9+6)**

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