

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc. DEGREE EXAMINATION – PHYSICS

FIRST SEMESTER – NOVEMBER 2019

UMT 1301 – MATHEMATICS FOR PHYSICS

Date: 05-11-2019

Dept. No.

Max. : 100 Marks

Time: 09:00-12:00

Part A

Answer ALL questions:

(10 x 2 = 20)

1. Determine the n^{th} derivative of $\sin(ax + b)$.
2. Show that in the parabola $y^2 = 4ax$, the subnormal is a constant.
3. Expand $(1 - x)^{-2}$.
4. State D' Alembert's ratio test.
5. Evaluate $L(t^2 + 2t + 3)$.
6. Find $L^{-1}\left(\frac{s}{s^2+a^2}\right)$.
7. Define a symmetric matrix. Give an example.
8. Find the characteristic equation of the matrix $A = \begin{pmatrix} 2 & -1 \\ -8 & 4 \end{pmatrix}$.
9. Define rank correlation.
10. Define Poisson distribution.

Part B

Answer any FIVE questions:

(5 x 8 = 40)

11. Find the angle of intersection of the curves $r = a(1 + \cos \theta)$ and $r = b(1 - \cos \theta)$.
12. Find the maxima and minima of the function $2x^3 - 3x^2 - 36x + 10$.
13. Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$.
14. Sum the infinite series $\frac{1.4}{5.10} - \frac{1.4.7}{5.10.15} + \frac{1.4.7.10}{5.10.15.20} - \dots$.
15. Find the Laplace transform of the periodic function $f(t) = \begin{cases} 1, & 0 < t < b \\ -1, & b < t < 2b \end{cases}$.
16. Verify Cayley - Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$.
17. Solve the following system of equations using Cramer's rule.
$$\begin{aligned} 2x + 3y - z &= 5 \\ 4x + 4y - 3z &= 3 \\ 2x - 3y + 2z &= 2 \end{aligned}$$
18. Calculate the mean and standard deviation for the following table giving the age distribution of 542 members.

Age in years	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of members	3	61	132	153	140	51	2

Part C

Answer any TWO questions:

(2 x 20 = 40)

19. (a) If $y = \sin(m \sin^{-1} x)$, prove that $(1 - x^2)y_2 - xy_1 + m^2y = 0$. Also prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.
 (b) Find the maximum value of $\int \frac{\log x}{x}$ for positive values of x . (12+8)
20. (a) Solve the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 4e^{-t}$ given $y(0) = 0$ and $y'(0) = 0$ using Laplace transforms.
 (b) Evaluate $L^{-1}\left(\frac{s-3}{s^2+4s+13}\right)$. (12+8)
21. (a) Find the Eigen vectors of the matrix $A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$.
 (b) Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$. (10+10)
22. (a) Prove that $\log \sqrt{12} = 1 + \left(\frac{1}{2} + \frac{1}{3}\right)\frac{1}{4} + \left(\frac{1}{4} + \frac{1}{5}\right)\frac{1}{4^2} + \left(\frac{1}{6} + \frac{1}{7}\right)\frac{1}{4^3} + \dots$
 $\log \sqrt{12} = 1 + \left(\frac{1}{2} + \frac{1}{3}\right)\frac{1}{4} + \left(\frac{1}{4} + \frac{1}{5}\right)\frac{1}{4^2} + \left(\frac{1}{6} + \frac{1}{7}\right)\frac{1}{4^3} + \dots$
 (b) Calculate the correlation coefficient for the following data.

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

(10+10)
