LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2019**

UMT 1502 – CALCULUS

Date: 01-11-2019 Dept. No. Max. : 100 Marks

Time: 09:00-12:00

SECTION – A

Answer ALL questions **(10 ò 2 = 20)**

1. Find the *n* th derivative of $\frac{1}{ax+b}$.
2. Find the *n* th derivative of xe^{5x} using Leibnitz theorem.
3. Find the slope of the curve $r = e^{\theta}$ at $\theta = 0$.
4. Show that in the curve $r = a\theta$, the polar subtangent varies as the square of the radius vector and the polar subnormal is constant.
5. Evaluate $\int tan^2 x\, dx$.
6. If *f* is an even function, what is $\int_{-a}^{a} f(x)\, dx$?
7. Evaluate $\int_0^1 \int_0^1 (x+y) dx\, dy$.
8. Evaluate $\int_0^1 \int_0^1 \int_0^1 xyz\, dx\, dy\, dz$.
9. Show that Γ (*n*+1) = *n* Γ (*n*).
10. Define Beta function.

SECTION – B

Answer any FIVE questions. **(5 ò 8 = 40)**

11. Investigate the maximum and minimum value of $4x^2 + 6xy + 9y^2 - 8x - 24y + 4$.
12. Find the *n*th derivative of $\frac{x+1}{(2x-1)(2x+1)}$.
13. Find the angle of intersection of the curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$.
14. Find the radius of curvature at the point $(\frac{a}{4}, \frac{a}{4})$ to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$.

15. Evaluate $\int \frac{2x+3}{x^2+5x+7}\,dx.$

16. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx.$

17. By changing the order of integration, evaluate $\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dxdy.$

18. Express $\int_0^1 x^m(1-x^n)^p\,dx$ interms of Gamma function and evaluate the integral $\int_0^1 x^5(1-x^3)^{10}dx.$

SECTION – C

Answer any TWO questions **(2 ò 20 = 40)**

19.(a) Show that the maximum value of $x^2y^2z^2$ subject to the condition $x^2+y^2+z^2=a^2$ is $\left(\frac{a^2}{3}\right)^3$.

(b) Find the lengths of the subtangent and subnormal at (a,a) on the cissoid $y^2=\frac{x^3}{2a-x}$. **(10+10)**

20. (a) Find the equation of the evolute of the parabola $y^2=4ax$, where $x=at^2$ and $y=2at$.

(b) Evaluate $\int \frac{2\,sinx+\cos x}{5+8\cos x}\,dx.$ **(10+10)**

21. (a) If $I_n = \int_0^{\frac{\pi}{4}} tan^n\,x\,dx$ where n is a positive integer, show that $I_n = \frac{1}{n-1} - I_{n-2}$ and hence evaluate $\int_0^{\frac{\pi}{4}} tan^6\,x\,dx.$

(b) Evaluate $\iiint xyz\,dx\,dy\,dz$ taken through the positive octant of the sphere $x^2+y^2+z^2=1$. **(10+10)**

22. (a) Evaluate $\int_R \int_R (x-y)^4 e^{x+y}dxdy$, where R is the square with vertices $(1,0),(2,1),(1,2)$ and $(0,1)$.

(b) Show that $\beta(m,n) = \frac{\Gamma(n)\Gamma(m)}{\Gamma(m+n)}$. **(10+10)**

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