LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2022**

17/18UMT1MC02 – ANALYTICAL GEOMETRY OF 2D, TRIG. MATRICES

Date: 03-12-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

PART – A

Answer ALL questions **(10 x 2 = 20)**

1. Write the expansion of $\sin n\theta$.
2. If $x = \cos\theta + isin\,\theta$, find $x^n + \frac{1}{x^n}$.
3. Prove that $cosh^2x - sinh^2x = 1$.
4. Find the value of $Log(4 + 3i)$.
5. State Cayley-Hamilton theorem.
6. Find the eigen values of the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$.
7. Find the pole of the line $Ax + By + C = 0$ with respect to the parabola $y^2 = 4ax$.
8. Define conjugate diameters of an ellipse.
9. Write the standard form of the equation to the hyperbola and its asymptotes.
10. What is the center of the hyperbola $9x^2 - 16y^2 + 72x - 32y = 16$?

PART – B

Answer any FIVE questions **(5 x 8 = 40)**

11. Expand $\cos 6\theta$ in terms of $\sin\theta$.
12. Evaluate $\lim_{x\to 0}\left(\frac{\sin 2x - 2\sin x}{x^3}\right)$.
13. Separate into real and imaginary parts $tan^{-1}(x + iy)$.
14. Prove that $sinh^{-1}\,x = \log\,(x + \sqrt{x^2 + 1})$.
15. Verify the Cayley-Hamilton theorem for the matrix $\begin{pmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{pmatrix}$.
16. Find the eigen values of the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.
17. Prove that in an ellipse, the tangents at the extremities of a diameter are parallel to the chords bisected by the diameter.
18. Find the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.

PART – C

Answer any TWO questions **(2 x 20 = 40)**

19. (a). If $\tan(x+iy) = u+iv$, prove that $\frac{u}{v} = \frac{\sin 2x}{\sinh 2y}$.

(b). Expand $sin^3\theta\ cos^4\theta$ in terms of multiples of θ. **(10+10)**

20. (a). If $\log\sin(\theta+i\phi) = A+iB$, then show that $2e^{2A} = \cosh 2\phi - \cos 2\theta$ and $\tanh\phi = \tan\theta\tan B$.

(b). If $\cosh u = \sec\theta$, show that $u = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$. **(10+10)**

21. Diagonalize the matrix $\begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$. **(20)**

22. (a). Show that the locus of the intersection of the tangents to $y^2 = 4ax$ which intercepts a constant length d on the directrix is $(y^2-4ax)(x+a)^2 = d^2x^2$.

(b). Find the locus of the foot of the perpendiculars drawn from the pole to the tangents to the circle $r = 2a\cos\theta$.

(10+10)

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