LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2022**

18UMT5MC03 – LINEAR ALGEBRA

Date: 28-11-2022 Dept. No. Max. : 100 Marks
Time: 09:00 AM - 12:00 NOON

PART – A

Answer ALL the questions: **(10 × 2 = 20)**

1. Define dimension of a vector space.
2. Write about linearly independent vectors.
3. Determine norm of (1, 2, 3) $\in R^3$ under usual metric.
4. Define an inner product space.
5. State the triangle inequality.
6. Define linear transformation.
7. What is a characteristic vector?
8. Define range for $T \in A(V)$.
9. Write brief notes on invariant subspace.
10. State a characterization theorem for unitary transformation.

PART – B

Answer any FIVE of the following: **(5 × 8 = 40)**

11. State and prove Schwarz inequality.
12. Express (1, -2, 5) as a linear combination of the vectors (1, 1, 1), (1, 2, 3), (2, -2, 1) in R^3.
13. Show that $T: R^2 \rightarrow R^2$ defined by $(a+b)T = (a+b,\ a)$ is a vector space homomorphism.
14. Prove that $T \in A(V)$ is invertible if and only of T maps V onto V.
15. Let $T \in A(V)$ and $\lambda \in F$. Then prove that λ is an eigenvalue of T if and only if $\lambda I - T$ is singular.
16. If V is a finite dimensional vector space over F, show that T ∈ A(V) is invertible if and only if the constant term of the minimal polynomial of T is non-zero.
17. Let $T \in A(V)$ be Hermitian. Prove that all its eigen values are real.
18. If $T \in A(V)$ such that $(vT, v) = 0\ \forall\ v \in V$, then prove that $T = 0$.

PART – C

Answer any TWO of the following: **(2 × 20 = 40)**

19. a) If V is a vector space of finite dimension and is the direct sum of its subspaces U and W, then prove that dim V = dim U + dim W.

    b) If V is a finite dimensional vector space over F, then prove that $T \in A(V)$ is regular if and only if T maps V onto V. **(10 + 10)**

20. If U and V are vector spaces of dimensions m and n respectively over F, prove that $Hom(U, V)$ is of dimension mn. **(20)**

21. Apply the Gram – Schmidt orthonormalization process to the vectors (1,0,1), (1,3,1) and (3,2,1) to obtain an orthonormal basis for $R^{3.}$ **(20)**

22. a) Let V be a vector space of all polynomials of degree less than or equal to 3. Let D be the differential operator defined by $(a + bx + cx^2 + dx^3)D = b + 2cx + 3dx^2$. Determine the matrix of D in the basis $1, x, x^2, x^3$.

    b) If $(vT, vT) = (v, v)\ \forall\ v \in V$ then prove that T is unitary. **(10 + 10)**

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