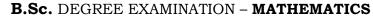
LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



FIFTH SEMESTER – **NOVEMBER 2022**

18UMT5MC03 – LINEAR ALGEBRA

Date: 28-11-2022 Dept. No. Time: 09:00 AM - 12:00 NOON

PART – A

Answer ALL the questions:

- 1. Define dimension of a vector space.
- 2. Write about linearly independent vectors.
- 3. Determine norm of $(1, 2, 3) \in \mathbb{R}^3$ under usual metric.
- 4. Define an inner product space.
- 5. State the triangle inequality.
- 6. Define linear transformation.
- 7. What is a characteristic vector?
- 8. Define range for $T \in A(V)$.
- 9. Write brief notes on invariant subspace.
- 10. State a characterization theorem for unitary transformation.

PART – B

Answer any FIVE of the following:

- 11. State and prove Schwarz inequality.
- 12. Express (1, -2, 5) as a linear combination of the vectors (1, 1, 1), (1, 2, 3), (2, -2, 1) in \mathbb{R}^3 .
- 13. Show that $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by (a + b)T = (a + b, a) is a vector space homomorphism.
- 14. Prove that $T \in A(V)$ is invertible if and only of T maps V onto V.
- 15. Let $T \in A(V)$ and $\lambda \in F$. Then prove that λ is an eigenvalue of T if and only if $\lambda I T$ is singular.
- 16. If V is a finite dimensional vector space over F, show that $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial of T is non-zero.

17. Let $T \in A(V)$ be Hermitian. Prove that all its eigen values are real.

18. If $T \in A(V)$ such that $(vT, v) = 0 \forall v \in V$, then prove that T = 0.



 $(10 \times 2 = 20)$

Max.: 100 Marks

 $(5 \times 8 = 40)$

PART – C

Answer any TWO of the following:

19. a) If V is a vector space of finite dimension and is the direct sum of its subspaces U and W, then prove that dim V = dim U + dim W.
b) If V is a finite dimensional vector space over F, then prove that T ∈ A(V) is regular if and only if T

maps V onto V.

- 20. If U and V are vector spaces of dimensions m and n respectively over F, prove that Hom(U,V) is of dimension mn.(20)
- 21. Apply the Gram Schmidt orthonormalization process to the vectors (1,0,1), (1,3,1) and (3,2,1) to obtain an orthonormal basis for R³.
 (20)
- 22. a) Let V be a vector space of all polynomials of degree less than or equal to 3. Let D be the differential operator defined by $(a + bx + cx^2 + dx^3)D = b + 2cx + 3dx^2$. Determine the matrix of D in the basis $1, x, x^2, x^3$.

b) If $(vT, vT) = (v, v) \forall v \in V$ then prove that T is unitary. (10 + 10)

aaaaaa

 $(2 \times 20 = 40)$

(10 + 10)