LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2022**

PMT1MC01 – LINEAR ALGEBRA

Date: 23-11-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	SECTION - A		
	Answer ALL the Questions		
1	**Answer the following** **(5 x 1 = 5)**		
a)	Define diagonalizable operator.	K1	CO1
b)	Write the eigen values of a projection operator.	K1	CO1
c)	Give an example for a nilpotent operator.	K1	CO1
d)	Write an inner product on R^3	K1	CO1
e)	Define a linear functional.	K1	CO1
2	**Multiple Choice Questions** **(5 x 1 = 5)**		
a)	Let A be a real nilpotent matrix of order 3 . Then the eigen values of A are a) 0, 0, 1 b) 0, 0, 0 c) 1, 1, 1 d) 1, 2, 3	K2	CO1
b)	A linear operator E with the property $E^2 = E$ called a) idempotent. b) nilpotent c) projection d) annihilator	K2	CO1
c)	A linear operator has distinct eigen values then it is a) not diagonalizable b) diagonalizable c) nilpotent d) zero matrix	K2	CO1
d)	Trace of a Elementary Jordan matrix of order 4×4 with characteristic value is 2 a) 2 b) 4 c) 6 d) 8	K2	CO1
e)	In R^2 $(\alpha\|\beta) = ax_1y_1 + bx_2y_2$ where $\alpha = (x_1, x_2), \beta = (y_1, xy_2)$ is an inner product if a) $a = 1, b = -2$ b) $a = 6, b = 0$ c) $a = 4 , b = 5$ d) For any real $a \; and \; b$.	K2	CO1
	SECTION - B		
	Answer any THREE of the following. **(3 x 10 = 30)**		
3	Write about the matrix of a projection operator.	K3	CO2
4	Let V be a finite-dimensional vector space over the field F and let T be a linear operator on V. Then prove that T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F.	K3	CO2
5	Write any four properties of nilpotent operators.	K3	CO2
6	Let V be a finite-dimensional vector space. Let $W_1, \ldots, W_k$ be subspaces of V and let $W = W_1 + \ldots + W_k$. Show that the following are equivalent. (i) $W_1, \ldots, W_k$ are independent. (ii) For each j, $2 \leq j \leq k$, $W_j \cap (W_1 + \ldots + W_{j-1}) = \{0\}$. (iii) If β_i is an ordered basis for W_i, $1 \leq i \leq k$, then the sequence $\beta = (\beta_1, \ldots, \beta_k)$ is an ordered basis for W.	K3	CO2
7	Write any four properties of an adjoint operator.	K3	CO2
	SECTION - C		
	Answer any TWO of the following. **(2 x 12.5 = 25)**		
8	Diagonalize the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$	K4	CO3
9	State and prove primary decomposition theorem.	K4	CO3

10	Let T be a linear operator on the finite-dimensional vector space V over the field F. Suppose that the minimal polynomial for T decomposes over F into a product of linear polynomials. Then prove that there is a diagonalizable operator D on V and a nilpotent operator N on V such that (i) $T = D + N$; (ii) $DN=ND$ Also show that the he diagonalizable operator D and the nilpotent operator N are uniquely determined by (i) and (ii) and each of them is a polynomial in T.	K4	CO3
11	Let T be a linear operator on R^2 which is represented by the matrix $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$. Let $\alpha = (0, 1)$. Show that $R^2 \neq Z(\alpha;\ T)$. Also prove that there is no non-zero β in R^2 with $Z(\beta;\ T)$ is disjoint from $Z(\alpha;\ T)$.	K4	CO3
	SECTION - D		
	Answer any ONE of the following. (1 x 15 = 15)		
12	If U is a linear operator on the finite dimensional space W, then U has a cyclic vector if and only if there is some ordered basis for W in which U is represented by the companion matrix of the minimal polynomial for U. Discuss about the rational form of a nilpotent transform.	K5	CO4
13	Prove the existence of cyclic decomposition theorem. Construct a linear transform and its cyclic vector on R^3	K5	CO4
	SECTION - E		
	Answer any ONE of the following. (1 x 20 = 20)		
14	Let F be a field and let B be an n x n matrix over F. Then B is similar over the field F to one and only one matrix which is in rational form. Create a transform and its rational form.	K6	CO5
15	a)Let V be a finite-dimensional inner product space, and f a linear functional on V. Then prove that there exists a unique vector β in V such that $f(\alpha) = (\alpha\|\beta)$ for all α in V. b)For any linear operator T on a finite-dimensional inner product space V, show that there exists a unique linear operator T^* on V such that $(T\alpha\|\beta) = (\alpha\|T^*\beta)$ for all α, β in V. Construct a non-identity operator T and its adjoint T^* on R^2	K6	CO5

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