LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2022**

PMT1MC03 – ORDINARY DIFFERENTIAL EQUATIONS

Date: 28-11-2022 Dept. No. Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

	SECTION A			
	Answer ALL the questions			
1	**Answer the following.** **(5 x 1 = 5)**			
a)	Describe the second order initial value problem.	K1	CO1
b)	Define linear dependence.	K1	CO1
c)	What is meant by fundamental matrix?	K1	CO1
d)	Define regular singular point.	K1	CO1
e)	Describe the non-oscillatory differential equation.	K1	CO1
2	**Choose the correct answer.** **(5 x 1 = 5)**		
a)	Let $f: [t_0, \infty] \rightarrow [0, \infty]$ be a continuous function and $k > 0$ be a constant. If $f(t) \leq k\int_{t_0}^{t} f(s)ds, t \geq t_0$, then which of the following holds? (a) $f(t) > 0$ (b) $f(t) < 0$ (c) $f(t) = 0$ (d) none of these	K2	CO1
b)	The Wronskian of $1, x$ and x^2 is (a) 1 (b) -1 (c) 2 (d) -2	K2	CO1
c)	A linear equation $x''' - 6x'' + 11x' - 6x = 0$ is transformed to linear system $x' = Ax$, where A is (a) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & 11 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & 11 & -6 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & 6 \end{bmatrix}$	K2	CO1
d)	When p is an integer, $J_{-P}(t) =$ (a) $J_P(t)$ (b) $pJ_P(t)$ (c) $(-1)^p J_P(t)$ (d) $-J_P(t)$	K2	CO1

e)	The equation $x'' - x = 0$ is (a) oscillatory (b) non-oscillatory (c) neither (a) nor (b) (d) both (a) and (b)	K2	CO1
	SECTION B		
	Answer any THREE of the following. (3 x 10 = 30)		
3	Apply Picard's successive approximation method to find the solution of the equation $x' = -x$, $x(0) = 1$, $t \geq 0$, and verify with analytical method.	K3	CO2
4	If the Wronskian of two functions x_1 and x_2 on I is non-zero for at least one point of I, show that x_1 and x_2 are linearly independent. Illustrate to $x_1 = t^2, x_2 = t\|t\|$ on $I = (-2,2)$.	K3	CO2
5	Consider a linear system $x' = A(t)x$ where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $A = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix}$. Show that $\Phi(\mathrm{t}) = \begin{bmatrix} e^{-3t} & te^{-3t} & t^2e^{-3t}/2 \\ 0 & e^{-3t} & te^{-3t} \\ 0 & 0 & e^{-3t} \end{bmatrix}$ is a fundamental matrix.	K3	CO2
6	Solve the equation $x'' - 2tx' + 2x = 0$.	K3	CO2
7	Demonstrate the comparison theorem in Sturm's perspective.	K3	CO2
	SECTION C		
	Answer any TWO of the following. (2 x 12.5 = 25)		
8	Let $b_1, b_2, \ldots, b_n: I \to \mathbb{R}$ be continuous functions in the n-th order homogeneous differential equation $L(x) = 0$. Let $\varphi_1, \varphi_2, \ldots, \varphi_n$ be n linearly independent solutions of $L(x) = 0$ on I. Obtain the Wronskian of $\varphi_1, \varphi_2, \ldots, \varphi_n$ and discuss the special case $L(x) = x''' + x'' + x' + x$.	K4	CO3
9	Derive the generating function and integral representation of Bessel function.	K4	CO3
10	Analyze the solutions of the system $x_1' = 5x_1 - 2x_2$ and $x_2' = 2x_1 + x_2$.	K4	CO3
11	Explain the Hille-Wintner comparison theorem.	K4	CO3
	SECTION D		
	Answer any ONE of the following. (1 x 15 = 15)		
12	Summarize the method of variation of parameters for solving the second order equation $x''(t) + b_1(t)x'(t) + b_2(t)x(t) = h(t)$ and implement to the particular case $b_1(t) = \frac{-2}{t}$, $b_2(t) = \frac{-2}{t^2}$, and $h(t) = tsint$.	K5	CO4
13	Let $x' = A(t)x$ be a linear system where $A: I \to M_n(R)$ is continuous. Suppose a matrix Φ satisfies the system, evaluate $(\det\Phi)'$ and assess that if Φ is a fundamental matrix if and only if $\det\Phi \neq 0$.	K5	CO4
	SECTION E		
	Answer any ONE of the following. (1 x 20 = 20)		
14	Formulate a unique solution for a class of initial value problem $x' = f(t, x)$ with $x(t_0) = x_0$ and discuss for the function $f(t, x) = t + x^2$, $x_0 = 0$.	K6	CO5

15	Develop the solution of the Legendre equation and its corresponding polynomial.	K6	CO5

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