	LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 03	34	
K	M.Sc. DEGREE EXAMINATION – MATHEMATICS		
R.	FIRST SEMESTER – NOVEMBER 2022		
Lu	PMT1MC06 – PROBABILITY THEORY AND RANDOM PROCESSI	ES	
F		100	
	ate: 02-12-2022 Dept. No. Max. : me: 01:00 PM - 04:00 PM	100	Marks
	SECTION A		
Ans	swer ALL the Questions		
1.		(5 x	1 = 5)
a)	State Bernoulli's law of large numbers.	K 1	CO1
b)	Identify the relation between the regression coefficients and correlation coefficient.	K1	CO1
c)	Tell the conditions for minimum variance unbiased estimators.	K1	CO1
d)	Define power of the test in statistical hypothesis.	K1	CO1
e)	Define Markov chain.	K1	CO1
2.		(5 x 1	= 5)
a)	If X and Y are independent, then identify which of the following is true?	K2	CO1
	(i) $E(XY) = E(X)E(Y)$		
	(ii) $E(XY) \le E(X)E(Y)$		
	(iii) $E(XY) \ge E(X)E(Y)$		
	(iv) None		
b)	Two variables x and y are related $y = ax + b$, where a and b are constants, then the	K2	CO1
	value of γ is		
	(i) +1 (ii) -1 (iii) ± 1 (iv) 0		
c)	If T is an unbiased estimator of θ , then T^2 is identified as	K2	CO1
	(i) Biased estimator of θ^2 (ii) Unbiased estimator of θ^2 (iii) Normal estimator of		
	θ^2 (iv) Binomial estimator of θ^2 .		
d)	The level of significance is estimated using the probability of	K2	CO1
	(i) Type I error (ii) Type II error		
	(iii) accepting H_0 when H_0 is false (iv) None		
e)	The mean of the random process is often expressed as	K2	CO1
	(i) Rank correlation (ii) Variance (iii) ensemble average (iv) auto-correlation		

				SE(CTIO	N B							
Ans	wer a	ny THREE of the follow	ving								(3)	x 10 =	= 30)
3.	State	e and prove weak law of l	arge n	umbers a	and he	ence ex	amine	wheth	ler the	law of		K3	CO2
		e numbers holds for the se	-								d		
		bllows: $P[X_k = \pm 2^k] = 2$											
4.		sulate the correlation coef						ther a	nd dau	ghter fr	·om	K3	CO2
		he following data:									-		
		eight of Mother (in Kgs.)	64	65	66	67	68	69	70	1			
		eight of Daughter (in	66	67	65	68	70	68	72	-			
	Kgs	s.)											
5.	A ra	ndom sample (X_1, X_2, X_3)) of siz	ze 3 is di	rawn f	rom a	norma	l popu	lation	with m	ean	K3	CO2
		d variance σ^2 . Let T_1, T_2									e μ,		
		re $T_1 = X_1 + X_2 - X_3, T_2$			$-4X_{2}$	and T_3	$_{3} = \frac{1}{3}($	<u>β</u> X ₁ +	- X ₂ +	<i>X</i> ₃).			
	(i) (ii)	Are T_1 and T_2 unbiased Calculate the value of β			o unhi	ased er	otimate	or for					
	` ´	ermine the best estimator.) Such	lliai 13 i	S uno.	.8300 0.	Stimue)] 101 [ι.				
6.	If x	\geq 1 is the critical region f	or test	$\operatorname{ing} H_0$: ($\theta = 2$	agains	t the al	ternat	ive H ₁	$: \theta = 1$, on	K3	CO2
		basis of the single observ				-							
		Jusis of the single observ	vation	from the	e popu	ulation,	, f(x,	$\theta) = \theta$	9 exp(- heta x),	≥ 0		
		∞ . Obtain the values of											
	<i>x</i> <		type										
7	x < signi	∞ . Obtain the values of ificance and power of the	type test.	I and ty	vpe II	errors.	. Also,	, calcı	ilate tl	he leve	l of		
7.	x < signi	∞ . Obtain the values of ificance and power of the pose a communication system.	type test.	I and ty ansmits	the dig	errors.	Also, $\frac{1}{1}$ Also, $\frac{1}{1}$, calcu	ilate th	he leve	l of . At		
7.	x < signi Supp each	∞ . Obtain the values of ificance and power of the pose a communication system stage the probability that	type test. tem transition	I and ty ansmits same di	ype II the dig git wi	errors. gits 0 a	. Also, and 1 th received	, calcu nrough d by t	ilate th many the ne:	he leve v stages xt stage	l of . At e as		
7.	x < signi Supp each trans	∞. Obtain the values of ificance and power of the pose a communication system stage the probability the smitted is 0.75. Using Chemical content of the probability of the probability the smitted is 0.75. Using Chemical content of the probability of the probability the smitted is 0.75. Using Chemical content of the probability of the	type test. stem tr at the apman	I and ty ansmits same di n-Kolmo	the dia git wi	errors. gits 0 a Ill be ro	. Also, and 1 th received	, calcu nrough d by t	ilate th many the ne:	he leve v stages xt stage	l of . At e as		
7.	x < signi Supp each trans	∞ . Obtain the values of ificance and power of the pose a communication system stage the probability that	type test. stem tr at the apman	I and ty ansmits same di n-Kolmo ering 0 a	the dia git wi	errors. gits 0 a Ill be ru v equati first sta	. Also, and 1 th received	, calcu nrough d by t	ilate th many the ne:	he leve v stages xt stage	l of . At e as		
	x < signi Supp each trans of re	∞. Obtain the values of ificance and power of the pose a communication system stage the probability the smitted is 0.75. Using Chemical content of the probability of the probability the smitted is 0.75. Using Chemical content of the probability of the probability the smitted is 0.75. Using Chemical content of the probability of the	type test. tem tra the at the apman by ente	I and ty ansmits same di n-Kolmo ering 0 a	the dig git wi ogorov at the f	errors. gits 0 a Ill be ru v equati first sta	. Also, and 1 th received	, calcu nrough d by t	ilate th many the ne:	he leve v stages xt stage	l of . At e as lity	12.5	= 25)
	x < signi Supp each trans of re	∞. Obtain the values of ificance and power of the pose a communication systemate the probability the smitted is 0.75. Using Checeiving 0 in the 5 th stage	type test. tem tra the at the apman by ente	I and ty ansmits same di n-Kolmo ering 0 a SEC	the dig git wi ogorov at the f	errors. gits 0 a ill be ru girst sta N C	. Also, and 1 th ecceived ions, ca	, calcu nrough d by t alcula	and the second s	he leve v stages. xt stage probabi	l of At as lity (2 x	12.5 K4	
Ans 8.	x < signi Supp each trans of re	∞. Obtain the values of ificance and power of the pose a communication systemate a communication systemate and probability that smitted is 0.75. Using Characteristic of the following 0 in the 5 th stage and prove Khintchin's inple.	type test. test. tem transition the state the theorem	I and ty ansmits same di n-Kolmo ering 0 a SEC m. In ac	ype II the dig git wi ogorov at the f C TIO ddition	errors. gits 0 a ill be ro equati first sta N C	. Also, and 1 th ecceived ions, ca age.	, calcu nrough d by t alcula applic	and the first of the second se	he leve v stages. xt stage probabi	l of At as lity (2 x	K4	CO3
Ans	x < signi Supp each trans of re	∞. Obtain the values of ificance and power of the pose a communication systemate the probability the smitted is 0.75. Using Checeiving 0 in the 5 th stage for the following and prove Khintchin's for the following of the following of the following for the follow	type test. test. tem transition the state the theorem	I and ty ansmits same di n-Kolmo ering 0 a SEC m. In ac	ype II the dig git wi ogorov at the f C TIO ddition	errors. gits 0 a ill be ro equati first sta N C	. Also, and 1 th ecceived ions, ca age.	, calcu nrough d by t alcula applic	and the first of the second se	he leve v stages. xt stage probabi	l of . At e as lity (2 x n an		CO3
Ans 8.	x < signi Supp each trans of re wer a State exan (a)	∞. Obtain the values of ificance and power of the pose a communication systematic stage the probability that smitted is 0.75. Using Characteristic of the following 0 in the 5 th stage my TWO of the followin e and prove Khintchin's inple. State and prove the invalue	type test. test. tem transmission the orem the orem the orem	I and ty ansmits same di n-Kolmo ering 0 a SEO m. In ac	ype II the dig git wi ogorov at the f CTIO	errors. gits 0 a ill be ro v equati first sta N C n, discu	. Also, and 1 th ecceived ions, ca age.	, calcu nrough d by t alcula applic mators	and the fill of th	he leve v stages. xt stage probabi through (5 mar	l of . At e as lity (2 x n an eks)	K4 K4	CO3 CO3
Ans 8.	x < signi Supp each trans of re	∞. Obtain the values of ificance and power of the pose a communication systematic stage the probability that smitted is 0.75. Using Characteristic of the following 0 in the 5 th stage my TWO of the followin e and prove Khintchin's inple. State and prove the invalues Compare the marks of	type test. test. tem transmission the the theorem theorem theorem	I and ty ansmits same di n-Kolmo ering 0 a SEC m. In ac e propert	the dig git wi ogorov at the f CTIO	errors. gits 0 a ill be ro equati first sta N C n, discu onsiste dents i	. Also, and 1 th ecceived ions, ca age.	, calcu nrough d by t alcula applic mators	and the fill of th	he leve v stages. xt stage probabi through (5 mar	l of . At e as lity (2 x n an eks)	K4	CO3
Ans 8.	x < signi Supp each trans of re wer a State exan (a)	∞. Obtain the values of ificance and power of the pose a communication systematic stage the probability that smitted is 0.75. Using Characteristic of the following 0 in the 5 th stage my TWO of the followin e and prove Khintchin's inple. State and prove the invalue	type test. test. tem transmission the the theorem theorem theorem	I and ty ansmits same di n-Kolmo ering 0 a SEC m. In ac e propert	the dig git wi ogorov at the f CTIO	errors. gits 0 a ill be ro equati first sta N C n, discu onsiste dents i	. Also, and 1 th ecceived ions, ca age.	, calcu nrough d by t alcula applic mators	and the fill of th	he leve v stages. xt stage probabi through (5 mar	l of . At e as lity (2 x n an eks)	K4 K4	CO3
Ans 8.	x < signi Supp each trans of re wer a State exan (a)	∞. Obtain the values of ificance and power of the pose a communication systematic stage the probability that smitted is 0.75. Using Characteristic of the following 0 in the 5 th stage my TWO of the followin e and prove Khintchin's inple. State and prove the invalues Compare the marks of	type test. test. tem trans at the apman by ente g theorem ariance otained ank co	I and ty ansmits same di n-Kolmo ering 0 a SEC m. In ac e propert	the dig git wi ogorov at the f CTIO	errors. gits 0 a ill be ro v equati first sta N C n, discu onsiste dents i hod.	. Also, and 1 th ecceived ions, ca ige. uss its int estin	, calcu nrough d by t alcula applic mators	and the fill of th	he leve v stages. xt stage probabi through (5 mar	l of . At e as lity (2 x n an eks)	K4 K4	CO3
Ans 8.	x < signi Supp each trans of re wer a State exan (a)	 ∞. Obtain the values of ificance and power of the pose a communication syster stage the probability that smitted is 0.75. Using Checeiving 0 in the 5th stage ny TWO of the followin e and prove Khintchin's mple. State and prove the invalues of Mathematics by using r 	type test. test. tem transmission the optimized theorem theorem optimized ank co	I and ty ansmits same di n-Kolmo ering 0 a SEC m. In ac e propert I by eigl orrelation	the dig git wi ogorov at the f CTIO	errors. gits 0 a ill be ro equati first sta N C n, discu onsiste dents i hod.	Also, and 1 the ecceived ions, ca age. uss its ant estin	, calcu nrough d by t alcula applic mators	and the first of t	he leve stages. xt stage probabi through (5 mar hysics	l of . At e as lity (2 x n an eks)	K4 K4	CO3

	_						, (0),	I I	at $T_1 =$	K4	
	T_2 , almost sur	ely.									
11.	A class of mo	dulated	signals	is mode	lled by the	e process	Y(t) = A	$X(t)\cos(\omega t)$	$(t + \theta),$	K4	CO
	where $X(t)$ i	s the n	iessage	signal v	which is a	a random	process	with mean	0 and		
	autocorrelation	n functio	on $R_{XX}($	τ). Acos	$(\omega t + \theta)$ i	is the carri	ier with a	mplitude A a	and the		
	frequency ω a	are cons	stants ar	nd the ini	itial carrie	er phase θ	is unifor	rmly distribu	uted in		
	$[-\pi,\pi]$. Also	X(t) an	d θ are	independ	ent. Show	v that $Y(t)$	is a WSS	S process.			
					SECTIO	ON D					
Ansv	wer any ONE	of the fo	ollowing	g					(1 x	x 15 =	15)
12.	State and prov	e Neym	an-Pear	son lemr	na.					K5	CO
13.	A panel of ju	dges A	and B	graded	seven deb	otors and	independ	ently award	ed the	K5	CO
	following mar	ks:									
	Debtors	1	2	3	4	5	6	7]		
	Marks of A	40	34	28	30	44	38	31			
	Marks of A Marks of B	40 32	34 39	28 26	30 30	44 38	38 34	31 28	-		
		32	39	26	30	38	34	28	sent. If		
	Marks of B	32 or was a	39 awarded	26 36 mark	30 s by judge	38 e A while	34 judge B	28 was not pres			
	Marks of B An eight debte judge B were	32 or was a also pre	39 awarded esent, ho	26 36 mark	30 s by judge marks wo	38 e A while ould you e	34 judge B	28 was not pres	ded to		
	Marks of B An eight debte	32 or was a also pre	39 awarded esent, ho	26 36 mark	30 s by judge marks wo	38 e A while ould you e	34 judge B	28 was not pres	ded to		
	Marks of B An eight debte judge B were the eighth de	32 or was a also pre	39 awarded esent, ho	26 36 mark	30 s by judge marks wo	38 e A while ould you e egree of	34 judge B	28 was not pres	ded to		
Ansv	Marks of B An eight debte judge B were the eighth de	32 or was a also pre btor as	39 awarded esent, ho suming	26 36 mark ow many that the	30 s by judge marks wo same de	38 e A while ould you e egree of	34 judge B	28 was not pres	ded to	x 20 =	: 20)
Ansv	Marks of B An eight debto judge B were the eighth de judgements?	32 or was a also pre btor as	39 awarded esent, ho suming	26 36 mark ow many that the	30 as by judge marks wo same de SECTIO	38 e A while ould you e egree of ON E	34 judge B except hin relationsh	28 was not pres	rded to n their (1	x 20 = K6	
Ansv 14.	Marks of B An eight debte judge B were the eighth de judgements? wer any ONE (a) A discret	32 or was a also pre btor as of the for ete varia	39 awarded esent, ho suming ollowing te <i>X</i> tak	26 36 mark ow many that the g es the va	30 30 30 30 30 30 30 30	38 e A while ould you e egree of ON E	34 judge B except hin relationsh ty 2 ^{-x} , x	28 was not present to be aware a second present to be aware a second present to be a sec	rded to n their (1 rove	1	= 20)
	Marks of B An eight debta judge B were the eighth de judgements? wer any ONE (a) A discrea that Che	32 or was a also pre btor as of the for ete varia	39 awarded esent, ho suming ollowing te <i>X</i> tak 's inequ	26 36 mark ow many that the g es the va	30 30 30 30 30 30 30 30	38 e A while ould you e egree of ON E	34 judge B except hin relationsh ty 2 ^{-x} , x	28 was not present to be aware a second present to be a se	rded to n their (1 rove	1	-
	Marks of B An eight debta judge B were the eighth de judgements? wer any ONE (a) A discrea that Che actual p	32 or was a also pre btor as of the for te varia byshev robabili	39 awarded esent, ho suming ollowing te <i>X</i> tak 's inequ ty.	26 36 mark ow many that the g es the va ality give	30 30 30 30 30 30 30 30	38 e A while ould you e egree of ON E 1 probabili $2 \ge 2) \le 1$	34 judge B except hin relationsh $ty \ 2^{-x}, x$ $\leq \frac{1}{2}. \text{ Also,}$	28 was not present to be awarting exists in $= 1, 2, \dots Pr$ determine it $(8 n)$	rded to n their (1 rove ts narks)	1	
	Marks of B An eight debto judge B were the eighth de judgements? wer any ONE of (a) A discrete that Che actual p (b) Explain	32 or was a also pre- btor as of the for ete varia ebyshev robability the deve	39 awarded esent, ho suming ollowing te <i>X</i> tak 's inequ ty.	26 36 mark ow many that the g es the va ality give	30 30 30 30 30 30 30 30	38 e A while ould you e egree of ON E 1 probabili $2 \ge 2) \le 1$	34 judge B except hin relationsh $ty \ 2^{-x}, x$ $\leq \frac{1}{2}. \text{ Also,}$	28 was not present to be awarting exists in the exist of the exist	rded to n their (1 rove ts narks) signal	1	
14.	Marks of B An eight debta judge B were the eighth de judgements? wer any ONE (a) A discre that Che actual p (b) Explain process.	32 or was a also pre btor as of the for the varia byshev robabili the devo	39 awarded esent, ho suming ollowing te X tak 's inequ ty. elopmer	26 36 mark ow many that the g es the va ality give	30 s by judge marks wo same de SECTIO lue <i>x</i> with es <i>P</i> (<i>X</i> – om proces	38 e A while ould you e egree of ON E 1 probabili $2 \ge 2) \le$ as in the ap	34 judge B except hin relationsh ty 2^{-x} , $x \le \frac{1}{2}$. Also,	28 was not present to be awarting exists in the exist of the exist	rded to n their (1 rove ts narks) signal narks)	K6	CO
	Marks of B An eight debto judge B were the eighth de judgements? wer any ONE of (a) A discrete that Che actual p (b) Explain	32 or was a also pre- btor as of the for ete varia byshev robabili the devo	39 awarded esent, ho suming ollowing te X tak 's inequ ty. elopmer	26 36 mark ow many that the g es the va ality give nt of rand physical	30 s by judge marks wo same de SECTIO lue <i>x</i> with es <i>P</i> (<i>X</i> – om proces	38 e A while ould you e egree of ON E probabili $2 \ge 2) \le$ as in the ap	34 judge B except hin relationsh ty 2^{-x} , $x \le \frac{1}{2}$. Also,	28 was not present to be awarting exists in the exist of the exist	rded to n their (1 rove ts narks) signal narks)	1	-