LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2022**

PMT 3501 – TOPOLOGY

Date: 21-11-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

Answer all Questions. All questions carry *equal* marks.

1. a. Define partially ordered set and totally ordered set with an example for each.

OR

b. What elements are maximal if the set $P = \{1,2,3,4,5\}$ if it is ordered as i) m$\leq n$ if m divides n ii) m$\leq n$ has the usual meaning. **(5)**

c. State and prove Cantor's intersection theorem. Give an example to show that the set F in cantor's intersection theorem may be empty if $d(F_n)$ does not converge to 0.

OR

d. Prove that the set $C(X,R)$ of all bounded continuous real functions defined on a metric space X is a real Banach space with respect to pointwise addition and scalar multiplication and the norm defined by $\|f\| = sup|f(x)|$. **(15)**

2. a. Prove or disprove the following:

i) If T_1 and T_2 are two topologies on a non-empty set X, $T_1 \cup T_2$is a topology on X.

ii) If T_1 and T_2 are two topologies on a non-empty set X, $T_1 \cap T_2$is a topology on X.

OR

b. Prove that any closed subspace of compact space is compact. **(5)**

c. Prove that every separable space is second countable.

OR

d. Define Lebesgue number of an open cover $\{G_i\}$of a metric space X. State and prove Lebesgue covering Lemma. **(15)**

3. a. Define T_1 and Hausdorff space with an example each.

OR

b. Prove that every compact subspace of a Hausdorff space is closed. **(5)**

c. State and prove Urysohn imbedding theorem.

OR

d. State and prove the Tietze Extension theorem. **(15)**

4. a. Define a connected space and give an example of connected and disconnected space.

OR

b. Prove that the components of a totally disconnected space are its points. **(5)**

c. Prove that the range of a continuous real function defined on a connected space is an interval.

OR

d. If X is an arbitrary topological space then prove the following;

i) each point in X is contained in exactly one component of X.

ii) each connected subspace of X is contained in a component of X.
iii) a connected subspace of X which is both open and closed is a component of X and
iv) each component of X is closed. **(15)**

5. a. Define locally connected and give an example of a locally connected space but not connected.

OR

b. Prove that a topological space X is disconnected if and only if there exists a continuous mapping of X onto the discrete two-point space $\{0,1\}$. **(5)**

c. State and prove the extended Stone Weierstrass theorem.

OR

d. Prove that any real continuous function defined on [a, b] can be approximated by a polynomial p(x) with real coefficients and $x \in [a, b]$ **(15)**

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