

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc. DEGREE EXAMINATION – PHYSICS

FIRST SEMESTER – NOVEMBER 2022

UMT 1301 – MATHEMATICS FOR PHYSICS

Date: 01-12-2022

Dept. No.

Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION - A

Answer ALL the Questions

1.	Answer the following		(5 x 1 = 5)
a)	State Leibnitz formula.	K1	CO1
b)	State D'Alembert's ratio test.	K1	CO1
c)	Define Laplace transform.	K1	CO1
d)	State Cayley-Hamilton theorem.	K1	CO1
e)	Define binomial distribution.	K1	CO1
2.	Fill in the blanks		(5 x 1 = 5)
a)	The formula to find the subtangent at a point P on the curve $y = f(x)$ is	K1	CO1
b)	The series expansion of e^x , for a real number x is	K1	CO1
c)	The inverse Laplace transform of $\frac{1}{s+2}$ is	K1	CO1
d)	The characteristic equation of a square matrix A is	K1	CO1
e)	The probability of getting a head on tossing a coin once is	K1	CO1
3.	Choose the correct answer		(5 x 1 = 5)
a)	The n^{th} derivative of e^{ax} is (i) e^{ax} (ii) ae^{ax} (iii) $a^n e^{ax}$ (iv) $a^n e^x$	K2	CO1
b)	If $ x < 1$, then the series $1 + x + x^2 + \dots$ converges to (i) $(1 + x)^{-1}$ (ii) $(1 - x)^{-1}$ (iii) $(1 - x)^{-2}$ (iv) $(1 + x)^{-2}$	K2	CO1
c)	What is $L\{\sinh 3t\}$? (i) $\frac{s}{s^2+9}$ (ii) $\frac{3}{s^2-9}$ (iii) $\frac{s}{s^2-9}$ (iv) $\frac{3}{s^2+9}$	K2	CO1
d)	The determinant of an $n \times n$ identity matrix is (i) n (ii) $2n$		

	(iii) n^2 (iv) 1	K2	CO1																
e)	The formula for the Poisson distribution function is (i) $\frac{e^{-\lambda} \lambda^x}{x!}$ (ii) $\frac{e^{-\lambda} \lambda^x}{x}$ (iii) $\frac{e^{\lambda} \lambda^{-x}}{x!}$ (iv) $\frac{e^{-\lambda} \lambda^x}{x}$	K2	CO1																
4.	Say True or False	(5 x 1 = 5)																	
a)	The minimum value of the function $f(x) = x^2$ is 0.	K2	CO1																
b)	The series $\sum_{n=1}^{\infty} \frac{1}{n^{0.5}}$ is convergent.	K2	CO1																
c)	The Laplace transform can be found for a function $f(t)$, for $t < 0$.	K2	CO1																
d)	The identity matrix of order 2 is $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.	K2	CO1																
e)	The normal distribution curve is bell shaped.	K2	CO1																
SECTION - B																			
Answer any TWO of the following in 100 words			(2 x 10 = 20)																
5.	Find the n^{th} derivative of $\frac{3}{(x+1)(2x-1)}$.	K3	CO2																
6.	Find $L^{-1} \left\{ \frac{1}{s(s+1)(s+2)} \right\}$.	K3	CO2																
7.	Find the inverse of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$.	K3	CO2																
8.	An urn contains 6 white, 4 red and 9 black balls. If 3 balls are drawn at random, find the probability that: (i) two of the balls are drawn white. (ii) one is of each colour. (iii) none is red. (iv) at least one is white.	K3	CO2																
SECTION - C																			
Answer any TWO of the following in 100 words			(2 x 10 = 20)																
9.	Find the maxima and minima of the function $2x^3 - 3x^2 - 36x + 10$.	K4	CO3																
10.	Test the convergence of the series $\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$	K4	CO3																
11.	Using Laplace transform evaluate $\int_0^{\infty} t e^{-3t} \cos t dt$.	K4	CO3																
12.	Calculate the correlation coefficient for the following data.	K4	CO3																
	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tbody> <tr> <td style="width: 10%;">X</td> <td>65</td> <td>66</td> <td>67</td> <td>67</td> <td>68</td> <td>69</td> <td>70</td> <td>72</td> </tr> <tr> <td>Y</td> <td>67</td> <td>68</td> <td>65</td> <td>68</td> <td>72</td> <td>72</td> <td>69</td> <td>71</td> </tr> </tbody> </table>			X	65	66	67	67	68	69	70	72	Y	67	68	65	68	72	72
X	65	66	67	67	68	69	70	72											
Y	67	68	65	68	72	72	69	71											

SECTION – D

Answer any ONE of the following in 250 words

(1 x 20 = 20)

13.	(a) If $y = e^{a \sin^{-1} x}$ prove that $(1 - x^2)y_2 - xy_1 - a^2y = 0$. Hence show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$. <p align="right">(10 Marks)</p>	K5	CO4																
	(b) Find the sum of the series $1 + \frac{3}{4} + \frac{3 \cdot 5}{4 \cdot 8} + \frac{3 \cdot 5 \cdot 7}{4 \cdot 8 \cdot 12} + \dots$ <p align="right">(10 Marks)</p>	K5	CO4																
14.	(a) Solve the following system of equations using Cramer's rule. $2x - y + 3z = 9; x + y + z = 6; x - y + z = 2$. <p align="right">(10 Marks)</p>	K5	CO4																
	(b) Calculate the mean for the following table giving the age distribution of 542 students. <table border="1" style="margin: 10px auto;"> <thead> <tr> <th>Age (in years)</th> <th>20-30</th> <th>30-40</th> <th>40-50</th> <th>50-60</th> <th>60-70</th> <th>70-80</th> <th>80-90</th> </tr> </thead> <tbody> <tr> <td>No. of members</td> <td>3</td> <td>61</td> <td>132</td> <td>153</td> <td>140</td> <td>51</td> <td>2</td> </tr> </tbody> </table> <p align="right">(10 Marks)</p>	Age (in years)	20-30	30-40	40-50	50-60	60-70	70-80	80-90	No. of members	3	61	132	153	140	51	2	K5	CO4
Age (in years)	20-30	30-40	40-50	50-60	60-70	70-80	80-90												
No. of members	3	61	132	153	140	51	2												

SECTION - E

Answer any ONE of the following in 250 words

(1 x 20 = 20)

15.	(a) Show that if $x > 0$, $\log x = \frac{x-1}{x+1} + \frac{1}{2} \frac{x^2-1}{(x+1)^2} + \frac{1}{3} \frac{x^3-1}{(x+1)^3} + \dots$ <p align="right">(10 Marks)</p>	K6	CO5
	(b) By using Laplace transform solve the differential equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 4e^{-4t}$, given that $y(0) = y'(0) = 0$. <p align="right">(10 Marks)</p>	K6	CO5
16.	Find the characteristic roots and associated characteristic vectors of the matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$. Is it possible to find the inverse of the matrix? Justify. <p align="right">(10 Marks)</p>	K6	CO5

@@@@@