LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2022**

UMT 1501 – ALGEBRA

Date: 24-11-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION - A			
Answer ALL the Questions			
1.	**Answer the following. (5 x 1 = 5 Marks)**		
a)	Identify the other roots of a biquadratic equation, when one of the roots is $\sqrt{5}-\sqrt{2}$.	K1	CO1
b)	State Descartes rule of sign.	K1	CO1
c)	Convert into partial fractions: $\frac{1}{(x-1)(x-2)}$.	K1	CO1
d)	State Cayley-Hamilton theorem.	K1	CO1
e)	Define Euler function.	K1	CO1
2.	**Fill in the blanks. (5 x 1 = 5 Marks)**		
a)	The product of the roots of the equation $x^4-5x^3+4x^2-6x+21=0$ is __________	K1	CO1
b)	The number of maximum possible positive real roots of the equation $x^5-2x^4-4x^3+5x^2-x+9=0$ is ___________.	K1	CO1
c)	The expansion of e^x+e^{-x} is ___________.	K1	CO1
d)	The characteristic equation of the matrix $A=\begin{bmatrix}3&0&0\\0&1&6\\0&0&-5\end{bmatrix}$ is _________________.	K1	CO1
e)	The formula for the product of all divisors of a number N is __________.	K1	CO1
3.	**Choose the correct answer for the following. (5 x 1 = 5 Marks)**		
a)	Every equation $f(x)=0$ with nth degree has _____. a. at least n-1 roots b. at most n+1 roots c. exactly n roots d. none of these	K2	CO1
b)	One of the positive roots of the equation $x^3-3x+1=0$ lies between a. 1 and 2 b. 2 and 3 c. 3 and 4 d. 4 and 5	K2	CO1
c)	The general term in the expansion of $(x+a)^n$ is______	K2	CO1

	a. $nC_r\, x^n\, a^r$ b. $nC_r\, x^{n-r}\, a^r$ c. $nC_r\, x^r\, a^n$ d. $nC_r\, x^r\, a^{n-r}$		
d)	The sum of the eigenvalues of the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ is a) 4 b) -6 c) 2 d) -2	K2	CO1
e)	If N is a prime number, then $\varphi(N)$ is a) 0 b) N c) N+1 d) N-1	K2	CO1
4.	**Say TRUE or FALSE. (5 x 1 = 5 Marks)**		
a)	If $f(a)$ and $f(b)$ are of like unlike signs, an odd number of roots of $f(x)$ lies between a and b.	K2	CO1
b)	Cardon's method is applicable for odd and even degree equations.	K2	CO1
c)	If $-1 < x < 1$, then $\log(1 + x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots$.	K2	CO1
d)	For every square matrix the sum and product of eigenvalues are always equal.	K2	CO1
e)	If $a \equiv b(mod\ m)$, then $a^n \equiv b^n(mod\ m)$.	K2	CO1
SECTION - B			
Answer any TWO of the following. (2 x 10 = 20 Marks)			
5	Solve the equation $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$ of which one root is $-1 + \sqrt{-1}$.	K3	CO2
6	Determine the sum of the fourth powers of the roots of the roots of the equation $x^5 - x^4 + 6x^2 - 4x + 5 = 0$.	K3	CO2
7	Apply exponential series to find the sum of the series $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \frac{1+3+3^2+3^3}{4!}$+… to ∞.	K3	CO2
8	Determine the sum and product of eigen values using characteristic equation for the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.	K3	CO2
SECTION - C			
Answer any TWO of the following. (2 x 10 = 20 Marks)			
9	Determine the roots of the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$.	K4	CO3
10	Resolve into partial fraction $\frac{x^2-10x+13}{(x-1)(x^2-5x+6)}$.	K4	CO3
11	Utilize the Cayley-Hamilton theorem, find the inverse for the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{bmatrix}$.	K4	CO3

12	Define amicable numbers. Also, show that 220 and 284 are amicable numbers.		K4	CO3
	SECTION - D			
Answer any ONE of the following.		**(1 x 20 = 20 Marks)**		
13		Using Horner's method, predict the positive root of $x^3 - 2x^2 - 3x - 4 = 0$ which lies between 3 and 4, correct to two decimal places.	K5	CO4
14	(a)	Prove Fermat's theorem and illustrate with an example. **(10 Marks)**	K5	CO4
	(b)	Determine the sum of the infinite series $\frac{15}{16} + \frac{15\cdot21}{16\cdot24} + \frac{15\cdot21\cdot27}{16\cdot24\cdot32} + \cdots$. **(10 Marks)**		
	SECTION - E			
Answer any ONE of the following.		**(1 x 20 = 20 Marks)**		
15	a)	Explain arithmetic progression and hence show that the roots of the equation $x^3 + px^2 + qx + r = 0$ are in arithmetical progression if $2p^3 - 9pq + 27r = 0$. Can you relate $x^3 - 6x^2 + 13x - 10 = 0$ with $x^3 + px^2 + qx + r = 0$? If the relation exists, then solve the $x^3 - 6x^2 + 13x - 10 = 0$. **(12 Marks)**	K6	CO5
	b)	If a, b, c denote three consecutive integers, formulate that $\log b = \frac{1}{2}\log a + \frac{1}{2}\log c + \frac{1}{2ac+1} + \frac{1}{3}\frac{1}{(2ac+1)^3} + \cdots$. **(8 Marks)**	K6	CO5
16	a)	Construct a 3×3 matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ from the roots of the following cubic equations. (i) The three roots of $x^3 - 5x^2 + 8x - 4 = 0$ are (a_{11}, a_{12}, a_{13}) where $a_{11} \leq a_{12} \leq a_{13}$. (ii) The three roots of $x^3 - 2x^2 + x = 0$ are (a_{21}, a_{22}, a_{23}) where $a_{21} \leq a_{22} \leq a_{23}$. (iii) The three roots of $x^3 - 4x^2 + 4x = 0$ are (a_{31}, a_{32}, a_{33}) where $a_{31} \leq a_{32} \leq a_{33}$. Also, verify Cayley-Hamilton theorem for the above constructed matrix. **(14 Marks)**	K6	CO5
	b)	Determine the number, sum and product of all the divisors of 360. **(6 Marks)**	K6	CO5

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