LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2022**

UMT 1502 – CALCULUS

Date: 03-12-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION A			
Answer ALL the Questions			
1.	**Answer the following: (5 x 1 = 5)**		
a)	State the Leibnitz formula for the derivative of the product of two functions.	K1	CO1
b)	Write the formula to find the angle between two curves in polar coordinates	K1	CO1
c)	State any two properties of definite integral.	K1	CO1
d)	State a result on Jacobian.	K1	CO1
e)	Write any two properties of beta function.	K1	CO1
2.	**Fill in the blanks (5 x 1 = 5)**		
a)	The nth derivative of $y = e^{3x}$ is _______________.	K1	CO1
b)	The slope of the curve $r = a(1 - cos\ \theta)$ at $\theta = \pi/2$ is _______________.	K1	CO1
c)	If f is an odd function, then $\int_{-a}^{a} f(x)dx$ is _______________.	K1	CO1
d)	If $f(x,y) = xy(x+y)$, then $\int_0^3 \int_1^2 f(x,y)dxdy$ is equal to _______________.	K1	CO1
e)	The value of $\Gamma(1/2)$ is _______________.	K1	CO1
3.	**Choose the correct answer for the following (5 x 1 = 5)**		
a)	Let $f: \mathbb{R} \to \mathbb{R}$ such that $f'(x) = 0$ and $f''(x) < 0$. Then at the point x, the function f is (i) increasing (ii) decreasing (iii) attains a maximum value (iv) attains a minimum value	K2	CO1
b)	If the curvature of a curve is $\frac{\pi}{6}$, then the radius of the curvature is (i) $\frac{\pi}{3}$ (ii) $\frac{\pi^2}{36}$ (iii) $\frac{6}{\pi}$ (iv) $\frac{\pi^2}{6}$	K2	CO1
c)	The $\int sin\ 3x\ dx$ is equal to (i) $\frac{1}{12}cos3x + \frac{3}{4}\sin x$ (ii) $\frac{-1}{3}cos3x$ (iii) $\frac{3}{4}\sin x$ (iv) $\frac{1}{12}sin3x + \frac{3}{4}\sin x$	K2	CO1
d)	The Jacobian of u, v with respect to x, y is denoted by (i) $J(\frac{u+v}{x+y})$ (ii) $J(\frac{u,v}{x,y})$ (iii) $J(\frac{xy}{uv})$ (iv) $J(\frac{x,y}{u,v})$	K2	CO1
e)	The value of $\beta(1,1)$ is (i) π (ii) 1 (iii) $\sqrt{\pi}$ (iv) 2	K2	CO1

4.	**Say TRUE or FALSE** **(5 x 1 = 5)**		
a)	The Lagrange's method of multipliers is used to find the maximum or minimum values of $f(x, y, z)$ subject to condition $\varphi(x, y, z) = 0$.	K2	CO1
b)	The formula to find the angle between two curves at (x,y) is $\theta = tan^{-1}(\frac{y}{x})$.	K2	CO1
c)	The value of $\int_0^{\pi} sinx\, dx$ is 2.	K2	CO1
d)	The Jacobian of x,y with respect to r, θ given $x = r\, cos\theta$ and $y = r\, sin\,\theta$ is 1.	K2	CO1
e)	Gamma function is said to be as Euler's integral of second kind.	K2	CO1
	SECTION B		
Answer any TWO of the following	**(2 x 10 = 20)**		
5.	Derive the n[th] derivative of $sin\, ax + e^{bx}$.	K3	CO2
6.	Prove that the subtangent for any point on the curve $y = be^{x/a}$ is of constant length and the subnormal is y^2/a.	K3	CO2
7.	Evaluate $\int \frac{3x+1}{(x-1)^2(x+2)} dx$.	K3	CO2
8.	Evaluate $\iint r\sqrt{(a^2 - r^2)}dr\, d\theta$ over the upper half of the circle $r = a\, cos\,\theta$.	K3	CO2
	SECTION C		
Answer any TWO of the following	**(2 x 10 = 20)**		
9.	Find the coordinates of the centre of curvature of the curve $y = x^2$ at the point (1/2, 1/4).	K4	CO3
10.	Evaluate (i) $\int_0^{\frac{\pi}{2}} \log sinx\;\; dx$. (ii) $\int x^n logx\, dx$	K4	CO3
11.	By transforming into polar coordinates, evaluate $\iint \frac{x^2y^2}{x^2+y^2}\, dx\, dy$ over the annular region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ where $b > a$.	K4	CO3
12.	(a) Show that $\Gamma(n + 1/2) = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n}\sqrt{\pi}$. (b) Evaluate $\int x^7(1-x)^8\, dx$.	K4	CO3
	SECTION D		
Answer any ONE of the following	**(1 x 20 = 20)**		
13.	(a) If $y = sin^{-1}x$, prove that $(1-x^2)y_2 - xy_1 = 0$ and $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$. **(10 marks)** (b) Evaluate the minimum value of $u = x^2 + y^2 + z^2$ when $x + y + z = 3a$. **(10 marks)**	K5	CO4
14.	(a) Show that the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$. **(13 marks)** (b) Prove that $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + cos^{\frac{3}{2}} x} dx = \frac{\pi}{4}$. **(7 marks)**	K5 K5	CO4 CO4

SECTION E			
Answer any ONE of the following		**(1 x 20 = 20)**	
15.	(a) Establish a reduction formula for $\int sin^m x\, cos^n x\ dx$, where m, n are positive integers. **(7 marks)**	K6	CO5
	(b) Evaluate $\iiint \frac{dx\, dy\, dz}{(x+y+z+1)^3}$ taken over the volume bounded by the planes $x = 0, y = 0$, $z = 0$ and x + y + z = 1. **(13 marks)**	K6	CO5
16.	(a) Prove that the relation between Beta and Gamma functions is $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(15 marks)**	K6	CO5
	(b) Evaluate $\int x^m \left(\log\left(\frac{1}{x}\right)\right)^n dx$. **(5 marks)**	K6	CO5

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