LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – NOVEMBER 2022

UMT 1502 - CALCULUS

Date: 03-12-2022 Time: 01:00 PM - 04:00 PM

Dept. No.

Max.: 100 Marks

SECTION A Answer ALL the Questions Answer the following: $(5 \times 1 = 5)$ 1. State the Leibnitz formula for the derivative of the product of two functions. K1 CO1 a) Write the formula to find the angle between two curves in polar coordinates K1 CO1 b) State any two properties of definite integral. K1 CO1 c) State a result on Jacobian. K1 CO1 d) Write any two properties of beta function. K1 CO1 e) $(5 \times 1 = 5)$ 2. Fill in the blanks The nth derivative of $y = e^{3x}$ is K1 CO1 a) The slope of the curve $r = a(1 - \cos \theta)$ at $\theta = \pi/2$ is _____ K1 b) CO1 If f is an odd function, then $\int_{-a}^{a} f(x) dx$ is _____. K1 CO1 c) If f(x, y) = xy(x + y), then $\int_0^3 \int_1^2 f(x, y) dx dy$ is equal to _____. K1 CO1 d) The value of $\Gamma(1/2)$ is _____ CO1 K1 e) 3. Choose the correct answer for the following $(5 \times 1 = 5)$ Let $f: \mathbb{R} \to \mathbb{R}$ such that f'(x) = 0 and f''(x) < 0. Then at the point x, the function f is K2 CO1 (i) increasing (ii) decreasing (iii) attains a maximum value (iv) attains a minimum a) value If the curvature of a curve is $\frac{\pi}{6}$, then the radius of the curvature is K2 CO1 b) (ii) $\frac{\pi^2}{36}$ (iii) $\frac{6}{\pi}$ (iv) $\frac{\pi^2}{6}$ (i) $\frac{\pi}{3}$ CO1 K2 The $\int \sin 3x \, dx$ is equal to c) (i) $\frac{1}{12}\cos 3x + \frac{3}{4}\sin x$ (ii) $\frac{-1}{3}\cos 3x$ (iii) $\frac{3}{4}\sin x$ (iv) $\frac{1}{12}\sin 3x + \frac{3}{4}\sin x$ The Jacobian of u, v with respect to x, y is denoted by K2 CO1 d) (i) $J(\frac{u+v}{x+y})$ (ii) $J(\frac{u,v}{x,y})$ (iii) $J(\frac{xy}{uv})$ (iv) $J(\frac{x,y}{u,v})$ The value of $\beta(1,1)$ is CO1 **K**2 e) (ii) 1 (iii) $\sqrt{\pi}$ (i) π (iv) 2

The Lagrange's method of multipliers is used to find the maximum or minimum valuesK2CO1a)of $f(x, y, z)$ subject to condition $\varphi(x, y, z) = 0$.K2CO1b)The formula to find the angle between two curves at (x, y) is $\theta = tan^{-1}(\frac{y}{z})$.K2CO1c)The value of $\int_{0}^{\pi} sinx dx$ is 2.K2CO1d)The Jacobian of x, y with respect to r, θ given $x = r \cos\theta$ and $y = r \sin\theta$ is 1.K2CO1e)Gamma function is said to be as Euler's integral of second kind.K2CO1c)Gamma function is said to be as Euler's integral of second kind.K2CO15.Derive then "derivative of $sin ax + e^{bx}$.K3CO26.Prove that the subtangent for any point on the curve $y = be^{x/a}$ is of constant length andK3CO28.Evaluate $\int \frac{x^{3x+1}}{(x-1)^{3}(x+2)} dx$.K3CO2SECTION CAnswer any TWO of the following(2 x 10 = 20)9.Find the coordinates of the centre of curvature of the curve $y = x^2$ at the point $(1/2, \mathbb{K}^4)$ CO31/4).10.EvaluateK4CO3(i) $\int_{0}^{\pi} \log sinx dx$. ((i) $\int_{0}^{\pi} \log sinx dx$. ((ii)K4CO31/4).Iby transforming into polar coordinates, evaluate $\iint \frac{x^2y^2}{x^2+y^2} dx dy$ over the annular region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ where $b > a$.K4CO312.(a) Show that $\Gamma(n + 1/2) = \frac{1\cdot3\cdot5\cdots(2n-1)}{2^n}\sqrt{n}$. (b) Evaluate $\int x^7(1 - x)^8 dx$.SECTION DSECTION D <td< th=""></td<>				
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13. $\frac{(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0.}{(10 \text{ marks})}$ K5 CO4				
13. (b) Evaluate the minimum value of $u = x^2 + y^2 + z^2$ when $x + y + z = 3a$. K5 CO4				
(10 marks)				
(a) Show that the evolute of the ellipse $\frac{x^2}{x^2} + \frac{y^2}{x^2} = 1$ is $(ax)^{2/3} + (by)^{2/3} =$				
$(a^2 - b^2)^{2/3}$. (13 marks)				
(b) Prove that $\int_{-\infty}^{\frac{\pi}{2}} \frac{\sin^2 x}{\cos^2 x} dx = \frac{\pi}{2}$. (7 marks) (7 marks)				
$\int_{0}^{1} \sin^{\frac{1}{2}x + \cos^{\frac{1}{2}x}} 4$				

SECTION E				
Answer any ONE of the following		$(1 \times 20 = 20)$		
15.	(a) Establish a reduction formula for $\int \sin^m x \cos^n x dx$, where m, n are positive integers	_{5.} K6	CO5	
	(7 marks)			
	(b) Evaluate $\iiint \frac{dx dy dz}{(x+y+z+1)^3}$ taken over the volume bounded by the planes $x = 0, y = 0$,	K6	CO5	
	z = 0 and x + y + z = 1. (13 marks))		
16.	(a) Prove that the relation between Beta and Gamma functions is $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$	<u>.</u> K6	CO5	
	(15 marks)		
	(b) Evaluate $\int x^m \left(\log\left(\frac{1}{x}\right) \right)^n dx.$ (5 marks)	K6	CO5	
