

LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034



B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – NOVEMBER 2022

UMT 1502 – CALCULUS

Date: 03-12-2022

Dept. No.

Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

PART – A

Answer all the questions:

(10 × 2 = 20)

1. Find nth derivative of e^x .
2. State Leibnitz formula for the derivative of the product of two functions.
3. Write the formula to find the angle between the radius vector and the tangent to a curve at given point.
4. Define Curvature of a curve.
5. Evaluate $\int e^{3x+4} dx$.
6. State any two properties of definite integral.
7. State a result on Jacobians.
8. Evaluate $\int_0^3 \int_0^2 xy \, dx dy$.
9. Define Gamma Integral.
10. State any two properties of beta function.

PART B

Answer any Five of the following:

(5 × 8 = 40)

11. Calculate nth derivative of $x^2 e^{5x}$
12. If $y = a \cos(\log x) + b \sin(\log x)$ prove that $x^2 y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + 1)y_n = 0$.
13. Find the angle of intersection of cardioids $r = a(1 + \cos \theta)$, $r = b(1 - \cos \theta)$.
14. Prove that $\int_0^{\frac{\pi}{4}} \log(1 + \tan \theta) = \frac{\pi}{8} \log 2$.
15. Find a reduction formula for $\int \cos^n x \, dx$, where n is a positive integer.
16. By changing the order of integration, evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy \, dx dy$
17. Prove that $\Gamma(n+1) = n!$
18. Prove that $\beta(m, n) = 2 \int_0^{\frac{\pi}{2}} \sin^{2m-1} \theta \cos^{2n-1} \theta \, d\theta$

PART – C

Answer any Two of the following:

(2 × 20 = 40)

19. Using Lagranges multipliers, find the maximum and minimum values of

$$f(x,y,z)=x+y+z \text{ subject to } \frac{1}{x} + \frac{1}{y} + \frac{1}{z}=1.$$

20. Find the value of the integral $\int \int \int xyz \, dx dy dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

21. Find subtangent subnormal, tangent and normal at the point (a,a) on the cissoids $y^2 = \frac{x^3}{2a-x}$.

22. Derive the relationship between beta and gamma function.

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