LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2022**

UMT 5501 – REAL ANALYSIS - II

Date: 23-11-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART – A

Q. No Answer ALL Questions (10 x 2 = 20 Marks)

1 Define signum function.

2 Using limit theorems, prove that $\lim_{x\to 2}\left(\frac{x^3-4}{x^2+1}\right)=\frac{4}{5}$.

3 Define Lipschitz function.

4 Show that cosine function is continuous on $\mathbb{R}$.

5 If $f: I \to \mathbb{R}$ has a derivative at $c \in I$, then prove that f is continuous at c.

6 Using L'Hospital's rule, evaluate $\lim_{x\to\infty}(e^{-x}x^2)$.

7 Prove that every constant function on $[a, b]$ is Riemann integrable on $[a, b]$.

8 Define tagged partition of an interval $[a, b]$.

9 Define open set.

10 Define compact set.

PART – B

Answer any FIVE Questions (5 x 8 = 40 Marks)

11 (i) If $f: A \to \mathbb{R}$ and if c is a cluster point of A, then prove that f can have only one limit at c. **(4)**

(ii) Using Squeeze theorem of limit, prove that $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1$. **(4)**

12 State and prove uniform continuity theorem. **(8)**

13 Let I, J be intervals in $\mathbb{R}$, let $g: I \to \mathbb{R}$ and $f: J \to \mathbb{R}$ be functions such that $f(J) \subseteq I$, and let $c \in J$. If f is differentiable at c and g is differentiable at $f(c)$, then prove that the composite function $g \circ f$ is differentiable at c and $(g \circ f)'(c) = g'(f(c)).f'(c)$. **(8)**

14 State and prove the squeeze theorem for integration. **(8)**

15 If K is a compact subset of $\mathbb{R}$ and $f: K \to \mathbb{R}$ is injective and continuous, then prove that f^{-1} is continuous on $f(K)$. **(8)**

16 State and prove Mean value theorem. **(8)**

17 If $f: [a, b] \to \mathbb{R}$ is continuous on $[a, b]$ then prove that $f \in \mathcal{R}[a, b]$. **(8)**

18 Prove that a subset of $\mathbb{R}$ is closed if and only if it contains all its cluster points. **(8)**

PART – C

Answer any TWO Question **(2 x 20 = 20 Marks)**

19 (a) Let $A \subseteq \mathbb{R}$,let f and g be functions on A to $\mathbb{R}$,and let $c \in \mathbb{R}$ be a cluster point of A.Further,let $b \in \mathbb{R}$. **(12)**
If $\lim_{x\to c} f = L$ and $\lim_{x\to c} g = M$,then prove that
(i) $\lim_{x\to c}(f + g) = L + M$, (ii) $\lim_{x\to c}(fg) = LM$, (iii) $\lim_{x\to c} bf = bL$ and
(iv) If $h: A \to \mathbb{R}$, if $h(x) \neq 0$ for all $x \in A$,and if $\lim_{x\to c} h = H \neq 0$,then prove that $\lim_{x\to c}\left(\frac{f}{h}\right) = \frac{L}{H}$.

(b) Let $f: A \to \mathbb{R}$ and let c be a cluster point of A.Prove that the following are equivalent **(8)**
(i) $\lim_{x\to c} f = L$.
(ii) For every sequence (x_n) in A that converges to c such that $x_n \neq c$ for all $n \in \mathbb{N}$, the sequence $(f(x_n))$ converges to L.

20 (a) Let $I = [a, b]$ and let $f : I \to \mathbb{R}$ be continuous on I.If $f(a) < 0 < f(b)$,or if $f(a) > 0 > f(b)$, then prove that there exists a number $c \in (a, b)$ such that $f(c) = 0$. **(14)**

(b) State and prove boundedness theorem. **(6)**

21 (a) Show that the union of an arbitrary collection of open subsets in $\mathbb{R}$ is open. **(5)**

(b) State and prove Additivity theorem of Riemann integrable. **(15)**

22 (a) If K is compact subset of R, then prove that K is closed and bounded. **(8)**

(b) State and prove Taylor's theorem. **(12)**

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