LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIFTH SEMESTER – **NOVEMBER 2022**

UMT 5601 – GRAPH THEORY

Date: 30-11-2022 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

SECTION – A

ANSWER ALL QUESTIONS: **(10 x 2 = 20)**

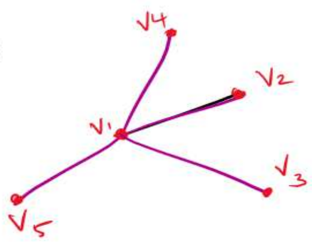
1. Differentiate complete and connected graphs.
2. Define Euler graph.
3. When a vertex is said to be incident and adjacent.
4. What is a Null graph? Give one example.
5. Define Hamiltonian path.
6. When a graph is said to be a Unicursal line?
7. A graph with atleast one vertex is also called a tree. True or False. Justify.
8. Prove that T is a tree if there is one and only path between every pair of vertices in a graph G.
9. Show that every bi-partite graph is $2-$ chromatic.
10. Define digraph.

SECTION – B

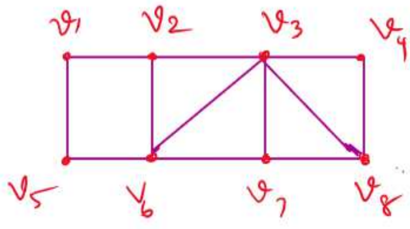
ANSWER ANY FIVE QUESTIONS: **(5 x 8 = 40)**

11. Prove “A graph G is disconnected if and only if its vertex set V can be partitioned into two non-empty, disjoint subsets v_1 and v_2 such that there exists no edge in G whose one end vertex is in subset v_1 and the other in subset v_2”.
12. Find the maximum and minimum degree of the following graphs:

(a)



(b)



13. If n is an odd number and $n \geq 3$, prove that in a complete graph with n vertices there are $(n-1)/2$ edge-disjoint Hamiltonian circuits.
14. A tree with n vertices has $n-1$ edges. – Justify.
15. Show that every circuit has a even number of edges in common without any cut set.
16. Prove that the vertex connectivity of a graph cannot exceed the edge connectivity of G.
17. Show that the complete bipartite graph $K_{3,3}$ is non-planar.
18. Prove that a graph with atleast one edge is $2-$ chromatic if and only if it has no cycles of odd length.

SECTION – C

ANSWER ANY TWO QUESTIONS: (2 x 20 = 40)

19. (a) Show that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.

    (b) Show that the number of vertices of odd degree in a graph G is always even with n vertices and e edges. **(15+5)**

20. (a) Prove that a connected graph G is an Euler graph if and only if all the vertices of G is even.

    (b) Show that a graph G with n vertices and n-1 edges and no cycles is connected.

    (10+10)

21. (a) Prove that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets.

    (b) Show that the maximum vertex connectivity of a graph G with n vertices and e edges is the integral part of $\frac{2e}{n}$. **(10+10)**

22. (a) State and prove Euler's formula.

    (b) Show that an $n-$ vertex graph is a tree iff its chromatic polynomial is $P_n(\lambda) = \lambda(\lambda - 1)^{n-1}$.

    (10+10)

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