LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **PHYSICS**

FIRSTSEMESTER – APRIL 2018

17/16PPH1MC04/PH 1820 - MATHEMATICAL PHYSICS - I

Date: 30-04-2018 Dept. No. Max. : 100 Marks
Time: 09:00-12:00

PART – A

Answer **ALL** the Questions (10x2=20)

1. Write the algorithm of Runge-Kutta method of solving differential equations.
2. Write down the expression associated with Euler's method.
3. Express $z = \frac{2+i}{1-i}$ in the form of $a + ib$.
4. State the condition for which the function is analytic.
5. Show that a nxn real antisymmetric tensor has $\frac{n(n-1)}{2}$ independent elements.
6. Define norm of a vector a and show that $(c, \alpha a + \beta b) = \alpha(c, a) + \beta(c, b)$
7. Show that $\delta_k^j . \delta_j^i = \delta_k^i$
8. Write the terms contained in the expression $G = g_{\mu\nu} x^\mu x^\nu$ for three dimensional space.
9. Define gamma function.
10. Sketch the graph for spherical Bessel's function.

PART-B

Answer any **FOUR** Questions (4x7.5=30)

11. Solve $x^3 + x^2 + 10x - 20 = 0$ using Regula Falsi method.
12. Evaluate $\int_c \frac{z^2 dz}{(4z+1)^2}$ where $c: |z| = 14$ using Cauchy's residue theorem.
13. Show that scalar product of two vector spaces (a,b) satisfies Cauchy-Schwarz inequality.
14. i) Prove that $A_{ij} B^i C^j$ is an invariant, if $B^i and\ C^j$ are contravariant vectors and A_{ij} is a covariant tensor
    ii) Prove that transformation of tensors form a group.
    iii) Show that, if a tensor is symmetric with respect to two indices in any coordinate system, it will remain symmetric with respect to these two indices in any other coordinate system.
15. i) Evaluate $\int_0^{\pi/2} sin^7\theta\ sin^8\theta\ d\theta$ using gamma and beta function.
    ii) Show that $J_{-\frac{1}{2}}(x) = \left(\frac{2}{\pi x}\right)^{1/2} \cos x$
16. Derive any two recurrence relations of Bessel's function.

PART-C

Answer any Four questions **(4x12.5=50)**

17. Apply Gauss-Seidal method to solve

$$5x + 2y + z = 12$$
$$x + 4y + 2z = 15$$

$x + 2y + 5z = 20$ Correct up to decimal places, taking $x_0 = y_0 = z_0 = 0$.

18. Using contour integration show that $\int_{-\infty}^{+\infty} \frac{x^2}{(1+x^2)^3} = \frac{\pi}{8}$

19. Diagonalize the matrix $A = \begin{vmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{vmatrix}$

20. i) If $A^{ij} and\ B^{ij}$ are two tensors, show that $A^{ij}B_{ij} = A_{ij}B^{ij}$

    ii) Show that in a Cartesian coordinate system, the contravariant and covariant components of a vector are identical.

    iii) Derive the components of Moment of inertia tensor.

21. Solve Legendre's differential equation by Frobenius power series method.

22. Solve by Gauss elimination method

$$6x - y - z = 19$$
$$3x + 4y + z = 26$$
$$x + 2y + 6z = 22$$

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