B.Sc.DEGREE EXAMINATION -PHYSICS

THIRD SEMESTER - APRIL 2018
PH 3506- MATHEMATICAL PHYSICS

Date: 05-05-2018
Time: 01:00-04:00
Dept. No. $\square$ Max. : 100 Marks

Part -A
Answer all questions
$(10 \times 2=20 \mathrm{marks})$

1. Define an analytic function
2. Separate the following into real and imaginary part of $\sin (x+i y)$
3. Find the unit normal to the surface $x^{2}+y^{2}=z$ at point $(1,2,5)$
4. State Stoke's theorem.
5. Define Euler coefficients for even half range expansion
6. Using Laplace integral, evaluate $\int_{0}^{\infty} \frac{\cos \omega d \omega}{1+\omega^{2}}$
7. What is a triangular matrix? Give an example
8. Determine the rank of a matrix $\left[\begin{array}{ccc}1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1\end{array}\right]$
9. Express Gauss' integral formula and give its importance.
10. Write down the difference operator for $f(x)$ by ' $h$ '.

## Part- B

Answer any four questions.
$(4 \times 7.5=30$ marks $)$
11. (a) Show that $|Z-i|^{2}=1$ describes a circle centered at the ( $0, i$ ) with radius 1 .
(b) Simplify $(1+\mathrm{i})(2+\mathrm{i})$ and locate it in the complex plane.
12. Using Green's theorem, evaluate $\int_{c}\left(x^{2} y d x+x^{2} d y\right)$ where C is boundary described counter clock wise of the triangle with vertices $(0,0)(1,0),(1,1)$.
13. Obtain a Fourier expression for $\mathrm{f}(\mathrm{x})=\mathrm{x}$ for $-\pi<x<\pi$.
14. Verify Cayley - Hamilton theorem for the matrix $\left(\begin{array}{ccc}1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1\end{array}\right)$ and find its inverse.
15. Using Lagrange's interpolation formula, find the value of $Y$ when $X=10$ from the following data.

| $X$ | 5 | 6 | 9 | 11 |
| :--- | :--- | :--- | :--- | :--- |
| $Y$ | 12 | 13 | 14 | 16 |

16. Use Cauchy's integral theorem to evaluate the integral $\oint_{C} \frac{d z}{z^{2}+1}$ where $C:|z+i|=1$ in the counter clockwise direction.

## Part -C

Answer any four questions. $(4 \times 12.5=50 \mathrm{marks})$
17. Establish that the real and complex part of an analytic function satisfies the Laplace equation.
18. (a) Prove that $\nabla . \nabla \times F=0$, where $F$ is a three dimensional vector in Cartesian coordinates.
(b) Using Gauss -divergence theorem, evaluate $\iint_{S}\left(x^{3} d y d z+y^{3} d z d x+z^{3} d x d y\right)$ where $S$ is the surface of the sphere $x^{2}+y^{2}+z^{2}=4$.
19. write down the functional form of a square wave of period $2 \pi$ and obtain its Fourier series.
20. Determine the eigen values of $A=\left[\begin{array}{ccc}2 & 0 & -2 \\ 0 & 0 & -2 \\ -2 & -2 & 1\end{array}\right]$ and show that matrix $A$ satisfies its own characteristic equation.
21. Calculate the approximate value of $\int_{-3}^{+3} x^{4} d x$ by Simpon's $\frac{1}{3}$ rd rule. Compare it with the exact value and the value obtained by Trapezoidal.
22. (a) Find the directional derivate of $g=\left(x^{2}+y^{2}+z^{2}\right)^{-1 / 2}$ at $(4,2,-4)$ in the direction of $(1,2,-2)$.
(b) If $\vec{U}=y z \hat{\imath}+z x \hat{\jmath}+x y \hat{k}$ and $\mathrm{f}=\mathrm{xyz}$, find $\operatorname{curl}(\mathrm{f} \vec{U})$

