LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

SECOND SEMESTER – **APRIL 2022**

UPH 2502 – MATHEMATICAL PHYSICS – I

(21 BATCH ONLY)

Date: 18-06-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

PART – A			
Q. No.	**Answer ALL Questions**		
1	**Select the right Choice** **5 x 1 = 5 Marks**		
(a)	The analytic function f(z) whose real part is x^2-y^2 (a) z (b) z^2 (c) z^3 (d) z^{-1}	K1	CO1
(b)	The function $f(z) = \frac{z}{z^2-1}$ in the contour C given by $x^2 + y^2 = 4$ (a) no pole (b) a simple pole at z = +1 (c) a simple pole at z = +1 & -1 (d) a simple pole at z = +i	K1	CO1
(c)	The value of triple product $\vec{a}\,.(\vec{a} \times \overrightarrow{b})$ is (a) zero (b) a simple pole z =2 (c) $\vec{a}$ (d) $\vec{b}$	K1	CO1
(d)	div $\vec{r}$ is (a) zero (b) 1 (c) 2 (d) 3	K1	CO1
(e)	The conditions imposed on function to be represented by Fourier series expansion is called (a) Parseval's condition (b) Dirichlet's (c) Euler's condition (d) Demorgan	K1	CO1
2	**Fill in the blanks** **5 x 1 = 5 Marks**		
(a)	The value of i^{178} is ………………	K1	CO1
(b)	If z = 1 - 7i then the value of imaginary part is …………………	K1	CO1
(c)	If vectors $\vec{\boldsymbol{a}}$ *and* $\vec{\boldsymbol{b}}$ are mutually perpendicular, then …………………	K1	CO1
(d)	$\vec{\imath}\,.\vec{\imath} =$ ………..	K1	CO1
(e)	If the function f(x) is odd, then f(-x) is equal to ……………………	K1	CO1
3	**Match the following** **5 x 1 = 5 Marks**		
(a)	Cauchy's integral theorem $\mid \vec{a}$ x $\vec{b}\mid$	K2	CO1
(b)	C-R Equations 0	K2	CO1
(c)	Area of the parallelogram Analytic	K2	CO1
(d)	Condition for coplanar $\int_{c} f(z)dz = 0$	K2	CO1
(e)	$\vec{k} \times \vec{k}$ $[\overrightarrow{a}\,\overrightarrow{b}\,\overrightarrow{c}] = 0$	K2	CO1
4	**True or False** **5 x 1 = 5 Marks**		
(a)	Let x + iy be a complex number and x – iy its complex conjugate.	K2	CO1
(b)	Let 1 + i be a complex number and its modulus 2.	K2	CO1

(c)	Curl of the vector field is always scalar.	K2	CO1
(d)	Gradient of the vector field is always scalar.	K2	CO1
(e)	For fourier representation of a function f(x), the function must be periodic.	K2	CO1

SECTION – B

Answer any TWO of the following (2 x 10 = 20)

5.	Show that the function $e^x(cosy + isiny)$ is an analytic function.	K3	CO2
6.	State and Prove Cauchy's Integral Theorem.	K3	CO2
7.	Show that $(y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational.	K3	CO2
8.	If $\vec{V} = \frac{x\hat{\imath}+y\hat{\jmath}+z\hat{k}}{\sqrt{x^2+y^2+z^2}}$, find the values of $div\,\vec{V}$.	K3	CO2

SECTION – C

Answer any TWO of the following (2 x 10 = 20)

9.	Prove that $U = x^2 - y^2$ and $V = \frac{y}{x^2+y^2}$ are harmonic functions of (x, y), but are not Harmonic conjugates.	K4	CO3
10.	Derive Cauchy-Riemann equations for a function to be analytic	K4	CO3
11	Find the values of a, b, c so that the function $\vec{f} = (x + 2y + az)\hat{\imath} + (bx - 3y - 3z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational	K4	CO3
12	Find the Fourier series to represent $f(x) = \pi - x \; for \;\; 0 < x < 2\pi.$	K4	CO3

SECTION – D

Answer any ONE of the following (1 x 20 = 20)

13	(a) Evaluate $\int_{c(} \frac{e^z}{z-1)(z-4)} dz$, Where '$c$' is the circle $\lvert z \rvert = 2$ by using Cauchy's integral formula. (b) Determine whether $\frac{1}{z}$ is analytic or not?	K5	CO4
14	(a) Prove that $[\vec{a} + \vec{b},\ \vec{b} + \vec{c}, \vec{c} + \overrightarrow{a}] = 2\,[\vec{a}\,\vec{b}\,\vec{c}]$ (b) Find the directional derivative of $x^2y^2z^2$at the point $(1,1,-1)$ in the direction of the tangent to the curve $x = e^t, y = sin2t + 1, z = 1 - cost$ at $t = 0$	K5	CO4

SECTION – E

Answer any ONE of the following (1 x 20 = 20)

15	Interpret the physical meaning of divergence and curl.	K6	CO5
16	An alternating current after passing through a rectifier has the form $i = I\, sin\theta \; for \; 0 < \theta < \pi$ $= 0 \qquad for \; \pi < \theta < 2\pi$, find the Fourier series of the function	K6	CO5

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