LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

THIRD SEMESTER – **NOVEMBER 2019**

16/17/18UPH3MC01 – MATHEMATICAL PHYSICS

Date: 29-10-2019 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART –A

Answer ALL Questions (10×2=20 marks)

1. Express $(1-\sqrt{2})+i$ in polar form.
2. Prove that (cosh x – sinh x)^n= cosh nx – sinh nx
3. What is meant by a solenoidal vector function?
4. If ¤ $= 3x^2y - y^3z^2$, find grad ¤ at the point (1,-2,-1).
5. What are even and odd functions?
6. Write the expression for Fourier sine integral.
7. Write any two assumptions made in deriving one dimensional wave equation for transverse vibration of the string.
8. Write the differential equation for one dimensional heat flow.
9. Using Trapezoidal rule, evaluate the missing ordinate

x	1	2	3	4	5
y	2	?	10	17	26

10. Define interpolation.

PART-B

Answer ANY **FOUR** Questions (4×7.5=30 marks)

11. (a) Derive Cauchy-Riemann equations in polar form.

    (b) Show that the function $e^x(\cos y + i\sin y)$ is an analytic function, find its first order derivative.

    (3+4.5)

12. Define directional derivative. Find the directional derivative of the function φ=x^2 - y^2+2z^2 at the point P (1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 1).
13. Using parity property of the function, obtain the Fourier series for the function f (x) = x in the interval -3 < x < 3.
14. Derive an expression for Fourier integral representing an aperiodic function.
15. Solve the following equation $\frac{\partial^2 U}{\partial x^2} - 2\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} = 0$ by the method of separation of variables.
16. (i). Discuss least square curve fitting for exponential function. From the table given below, find the best values of 'a' and 'b' in the law y= e^{bx} by the method of least squares.

x	0	5	8	12	20
y	3.0	1.5	1.0	0.55	0.18

PART-C

<u>Answer ANY **FOUR** Questions</u> (4×12.5=50 marks)

17. (a) State and prove Cauchy's integral theorem .Find the value of $\int \frac{z+4}{z^2+2z+5}dz$ if C is the circle

    $|Z+1|=1$

    (b) Evaluate $\left[\frac{1+\sin\sim+i\cos\sim}{1+\sin\sim-i\cos\sim}\right]^n$ (8+4.5 marks)

18. (a) State and prove Green's theorem (7marks).

    (b) Using Green's theorem, evaluate $\int_c (x^2 ydx + x^2 dy)$, where "c" is the boundary described counter clockwise of the triangle with vertices (0, 0), (1, 0), (1, 1) (5.5 marks).

19. (a) Obtain the Fourier transform of the function $f(x) = e^{-ax^2}\ ; a > 0$

    (b) Using the Fourier transform of derivatives, find the Fourier transform of $x\ e^{-ax^2}$

20. A rod of length "l" with insulated sides is initially at a uniform temperature 'u'. Its ends are suddenly cooled at 0°C and are kept at that temperature. Prove that the temperature function u(x,t) is given by

    $$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n®x}{l}e^{-\frac{C^2®^2n^2t}{l^2}}$$

21. (a) Derive Lagrangian interpolation formula. (5).

    (b) Derive Simpson's 1/3 rd rule. Using it, evaluate $\int_0^{®} \sin xdx$ by dividing the range into ten equal parts. Verify your answer with actual integration (7.5 marks).

22. Solve the equation $\frac{dy}{dx} = 1 - y$, given y(0)=0 using modified Euler's method and tabulate the solutions at x=0.1,0.2 and 0.3. Also get the solutions by improved Euler method.

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