LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **PHYSICS**

FIRST SEMESTER – **NOVEMBER 2022**

PPH1MC03 – MATHEMATICAL PHYSICS

Date: 28-11-2022 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

	SECTION A		
	Answer ALL the questions		
1	**State True or False** **(5 x 1 = 5)**		
a)	All harmonic functions are analytic.	K1	CO1
b)	All periodic functions have period of 2π.	K1	CO1
c)	The kernel of Fourier transform is e^{sx}.	K1	CO1
d)	All trigonometric functions are orthogonal to one another in the limit -1 to +1.	K1	CO1
e)	The sum of probability of failures and success is 1.	K1	CO1
2	**Answer the following** **(5 x 1 = 5)**		
a)	State Cauchy's integral formula.	K2	CO1
b)	What are mutually exclusive events?	K2	CO1
c)	State any two axioms of a vector space.	K2	CO1
d)	Write the recurrence formulae for Binomial distribution.	K2	CO1
e)	State first shifting theorem in Laplace transform.	K2	CO1
	SECTION B		
	Answer any THREE of the following in 500 words **(3 x 10 = 30)**		
3	Prove that $\Gamma\frac{1}{2} = (\pi)^{1/2}$ from first principles.	K3	CO3
4	State and prove Cauchy's theorem from first principles.	K3	CO3
5	Show that $u = e^{x}\sin y$ is harmonic. Construct $f(z) = u + iv$, such that $f(z)$ is an analytic function.	K3	CO3
6	(a) A manufacturer knows that the condensers he manufactures contain on an average of 1% of defective ones. If he packs them in boxes of 100 what is the probability that a box picked at random will contain 4 or more faulty condensers? (b) An insurance company found that 0.01% of the population is involved in a certain type of accident each year. If 1000 policy holders of the company were randomly selected, what is the probability that not more than two of its clients would be involved in such an accident the next year? (given that $e^{-0.1} = 0.9048$). (5+5)	K3	CO3

7	Evaluate $L^{-1}\left[\frac{1}{(s^2+2s+5)}\right]$	K3	CO3
	SECTION C		
	Answer any TWO of the following in 500 words **(2 x 12.5 = 25)**		
8	Examine the following system of vectors for linear dependence. If dependent, find the relation between them. a) $X_1 = (1, -1, 1), X_2 = (2, 1, 1), X_3 = (3, 0, 2)$. b) $X_1 = (3, 1, -4), X_2 = (2, 2, 1), X_3 = (0, -4, 1)$. c) $X_1 = (1, 1, 1, 3), X_2 = (1, 2, 3, 4), X_3 = (2, 3, 4, 7)$. d) $X_1 = (1, 1, -1, 1), X_2 = (1, -1, 2, -1), X_3 = (3, 1, 0, 1)$. e) $X_1 = (1, -1, 2, 0), X_2 = (2, 1, 1, 1), X_3 = (3, -1, 2, -1)$, $X_4 = (3, 0, 3, 1)$.	K4	CO3
9	Using Parseval's identity, evaluate $\int_0^\infty \frac{dx}{(x^2+1)^2}$	K4	CO3
10	Prove that $P_{n+1}(x) = \frac{2n+1}{n+1}\, x\, P_n(x) - \frac{n}{n+1}\, P_{n-1}(x)$. P's stand for Legendre Polynomials.	K4	CO3
11	(a) A student takes his examination in four subjects α, β, γ, δ. He estimates his chances of passing in α as $\frac{4}{5}$, in β as $\frac{3}{4}$, in γ as $\frac{5}{6}$ and in δ as $\frac{2}{3}$. To qualify, he must pass in α and at least in two other subjects. What is the probability that he qualifies? (b) The probability that a man aged 60 will live to be 70 is 0.65. What is the probability that out of 10 men, now 60, at least 7 will live to be 70?	K4	CO3
	SECTION D		
	Answer any ONE of the following in 1000 words **(1 x 15 = 15)**		
12	Evaluate $\int_0^{2\pi} \frac{4}{5-4\sin\theta}\, d\theta$.	K5	CO4
13	Find the inverse Fourier transform of $F\,(s) = e^{-\lvert s\rvert y}$	K5	C04
	SECTION E		
	Answer any ONE of the following in 1000 words **(1 x 20 = 20)**		
14	State and prove the orthogonality relation of Hermite polynomials.	K6	CO5
15	Evaluate $\int_{-\infty}^{\infty} \frac{x^2\, dx}{(x^2+1)(x^2+4)}$ using contour integration.	K6	CO5

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