# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**I M.Sc.** DEGREE EXAMINATION **STATISTICS**

SECOND SEMESTER – **APRIL 2015**

**ST 2814 - ESTIMATION THEORY**

**Time: 3 hours**
**Max : 100 marks**

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## PART – A

**Answer ALL the questions** **(10X2=20 marks)**

1. Give an example to prove that an unbiased estimator need not be unique.
2. Define UMVUE for estimating a parameter θ.
3. Define Sufficient Statistic and provide an example.
4. Find which one of the following is ancillary when a random sample $X_1$, $X_2$ is drawn from N(μ,1).
   (a) $X_1/X_2$ (b) $X_1+X_2$ (c) $X_1 - X_2$ (d) $2X_1-X_2$
5. Give an example of a family of distributions which is not complete.
6. Explain exponential class of family.
7. Suggest an MLE for P[X=0] in the case of Poisson (θ).
8. Let X~ B(1, θ) , θ = 0.1,0.2,0.3. Find MLE of θ.
9. Define CAN estimator.
10. Explain Jackknife method.

## PART – B

**Answer any FIVE questions** **(5X8=40 marks)**

11. Let X be a discrete r.v. with $P(x;\theta) = \begin{cases} \theta & , x=-1 \\ (1-\theta)^2\,\theta^x & , x=0,1,2,... \end{cases}$
    Find all the unbiased estimators of 0.
12. Obtain UMVUE of θ(1- θ) using a random sample of size n drawn from a Bernoullie population with parameter θ .
13. Let X~ N (θ,1). Obtain the Cramer- Rao lower bound for estimating $\theta^2$ . Compare the variance of the UMVUE with CRLB.
14. i) State and Establish Basu's theorem
    ii) Define UMRUE
15. If T is sufficient is for **P** or θ, then show that one-one function of T is also sufficient for **P** or θ. Illustrate with an example.
16. State and establish Lehmann-Scheffe theorem.
17. i. State Cramer-Rao regularity conditions
    ii. State and prove CR inequality.
18. Explain Bootstrap method with example.

## PART – C

**Answer any TWO questions** **(2 X 20 = 40marks)**

19. (a) Let $\delta_0$ be a fixed member of $U_g$. Prove that $U_g = \{\delta_0 + u | u \in U_0\}$ .

    (b) Let $X_1$,$X_2$ be a random sample with E(0,σ). Show that ($X_1$+$X_2$) and $X_1$|($X_1$+$X_2$) are independent using Basu's theorem. (10+10)

20. (a) If $\{\delta_n\}$ is a sequence of UMVUE's and $\delta_n \to \delta$ a.s as $n \to \infty$, then show that $\delta$ is UMVUE.

    (b) State and establish Uncorrelatedness approach of UMVUE. (10+10)

21. (a) Let ($X_i$,$Y_i$) , i=1,2,…,n be a random sample from ACBVE distribution with pdf

    $f(x,y) = \{(2\alpha+\beta)(\alpha+\beta)/2\}\exp\{-\alpha(x+y)-\beta\max(x,y)\}, \quad x,y>0.$

    Find MLE of α and β.

    (b) MLE is not consistent – Support the statement with an example. (10+10)

22. (a) "Blind use of Jackknifed method" – Illustrate with an example.

    (b) Explain Baye's estimation with an example. (10+10)

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