LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – STATISTICS

SECOND SEMESTER – **APRIL 2015**

ST 2815 - TESTING STATISTICAL HYPOTHESIS

Date : 18/04/2015 Dept. No. Max. : 100 Marks
Time : 01:00-04:00

SECTION – A

Answer ALL the following questions **(10 x 2 = 20)**

1. What is randomized test function?
2. Write any two examples for simple and composite hypothesis.
3. Define uniformly most powerful test.
4. Let X ~ B(1, θ), θ = 0.4,0.5. For testing H: θ = 0.4 Vs K: θ = 0.5, a test is given by
   $$\Phi(x) = \begin{cases} 0.2 & if \quad x = 0 \\ 0.5 & if \quad x = 1 \end{cases}$$
   Find the power of the test.
5. Show that an UMP test is unbiased.
6. Define MLR property.
7. Justify the following statement
   "A test with Neyman structure is α – similar"
8. Define multi-parameter exponential family with an example.
9. Show that $T(x) = \frac{X_1}{X_2}$ is invariant with respect to scale transformation.
10. Briefly explain the principles of LRT.

SECTION – B

Answer any FIVE of the following questions **(5x 8 = 40)**

11. Prove that one parameter exponential family possesses Monotone Likelihood Property.
12. Consider the following probability distribution of X.

X	1	2	3	4	5
H	0.02	0.03	0.015	0.05	0.885
K	0.03	0.045	0.15	0.24	0.535

Define three test functions $\varphi_1, \varphi_2 \text{ and } \varphi_3$ such that

$$\varphi_1 = \begin{cases} 1 & if\ x = 1,3,4 \\ 0 & , Otherwise \end{cases}, \ \varphi_2 = \begin{cases} 1 & if\ x = 3 \\ 0, & Otherwise \end{cases} \text{ and } \varphi_3 = \begin{cases} 1 & if\ x = 1\ or\ 2 \\ 0, & Otherwise \end{cases}.$$

Identify the most powerful test of level 0.05.

13. Let $X_1, X_2, \ldots, X_n$ be a random sample from U(0,θ),θ > 0. Derive UMP level α test for testing the hypothesis H : θ = θ_0 against K : θ ≠ θ_0.

14. Let X follows B(1,θ), $\theta = 0.1, 0.2, 0.3$. For testing the hypothesis $H : \theta = 0.2$ against $K : \theta = 0.1, 0.3$. Show that UMP test does not exist.

15. State the generalized Neyman - Pearson lemma.

16. Derive UMPU level α test for testing the hypothesis $H : \theta_1 \leq \theta \leq \theta_2$ against $K : \theta < \theta_1$ or $\theta > \theta_2$ for one parameter exponential family.

17. Let $X_1, X_2, \ldots, X_m$ be a random sample from P(λ) and $Y_1, Y_2, \ldots, Y_n$ be a random sample from P(μ) . Derive UMPU level α test for testing the hypothesis $H : \lambda \leq \mu$ Vs $K: \lambda > \mu$.

18. Define Multi Parameter exponential family and its properties.

SECTION – C

Answer any TWO of the following questions **(2x 20 = 40)**

19. State and prove the necessary as well as sufficient conditions of fundamental Neyman – Pearson lemma. **(20)**

20. a) State and prove MLR theorem. **(10)**

    b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$, $\theta \in R$. Derive UMP level α test for testing the hypothesis $H_0 : \mu \leq \mu_0$ against $H_1 : \mu > \mu_0$. . **(10)**

21. Consider the (k+1) parameter exponential family.
    Derive the conditional UMPU level α test for testing $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$. **(20)**

22. Let X and Y follows $N(\mu_1, \sigma_1^2)$ an $N(\mu_2, \sigma_2^2)$. Derive Likelihood ratio test for testing $H_0: \mu_1 = \mu_2 \; Vs \; H_0: \mu_1 \neq \mu_2$ when population variances are equal. **(20)**

****